Calc II - Review for exam I

The first exam will be this Friday, September 18. We will discuss some of these problems in class on Wednesday, but you should work them all out to the best of your ability prior to that. Understanding the problems on this sheet will help you greatly on the exam.

1. Evaluate the following integrals using the technique indicated.
   (a) $\displaystyle\int \frac{x}{2x^2+1}\,dx$ - u-subs
   (b) $\displaystyle\int x\cos(x)dx$ - by parts
   (c) $\displaystyle\int \cos^2(x)\,dx$ - with the $\frac{1}{2}$-angle formula
   (d) $\displaystyle\int_0^1 \frac{x^3}{\sqrt{1-x^2}}\,dx$ - trig subs
2. The complete graph of a function f is shown in the bottom of figure 1 on the reverse - it consists of two straight line segments and a quarter circle.
   (a) Evaluate $\displaystyle\int_0^5 f(x)\,dx$.
   (b) Write down the left Riemann sum with $n = 5$ terms representing $\displaystyle\int_0^5 f(x)\,dx$.
3. Consider the integral $\displaystyle\int_0^2 x\sin\left(\cos\left(x^2+1\right)\right)dx$. Letting $u = x^2+1$, translate this integral to a new integral in terms of u. You need not evaluate the integral.
4. Suppose we know that
$$\int_{-3}^{3} f(x)dx = 6.$$
   Which of the following integrals can we compute and what is the value?
   (a) $\displaystyle\int_{-1}^{1} f(3x)dx$
   (b) $\displaystyle\int_0^2 f(3x-3)dx$
   (c) $\displaystyle\int_0^3 f(x)dx$
5. (a) Use integration by parts to derive a reduction formula for the integral $\displaystyle\int x^n e^x dx$.
   (b) Use your reduction formula to evaluate $\displaystyle\int x^3 e^x dx$.
6. Suppose we want to estimate $\int_0^{10} e^{-x^2}\,dx$ using a right Riemann sum with 100 terms.
   (a) Write down the resulting Riemann sum using summation notation.
   (b) Is your estimate a lower bound or an upper bound?

7. Let

$$F(x) = \int_0^{x^3} \cos(t^2)dt.$$

Compute $F'(x)$.

8. Evaluate the following integrals using any technique that you see fit.

(a) $\displaystyle\int_0^2 \sqrt{4-x^2}\,dx$

(b) $\displaystyle\int x\sqrt{4-x^2}\,dx$

(c) $\displaystyle\int x^2\sqrt{4-x^2}\,dx$

(d) $\displaystyle\int_{-\pi}^{\pi} \frac{\sin^3(x)}{\cos^5(x)}\,dx$

(e) $\displaystyle\int e^{2x}\cos(3x)dx$

(f) $\displaystyle\int \frac{1}{x^2\sqrt{4-x^2}}\,dx$

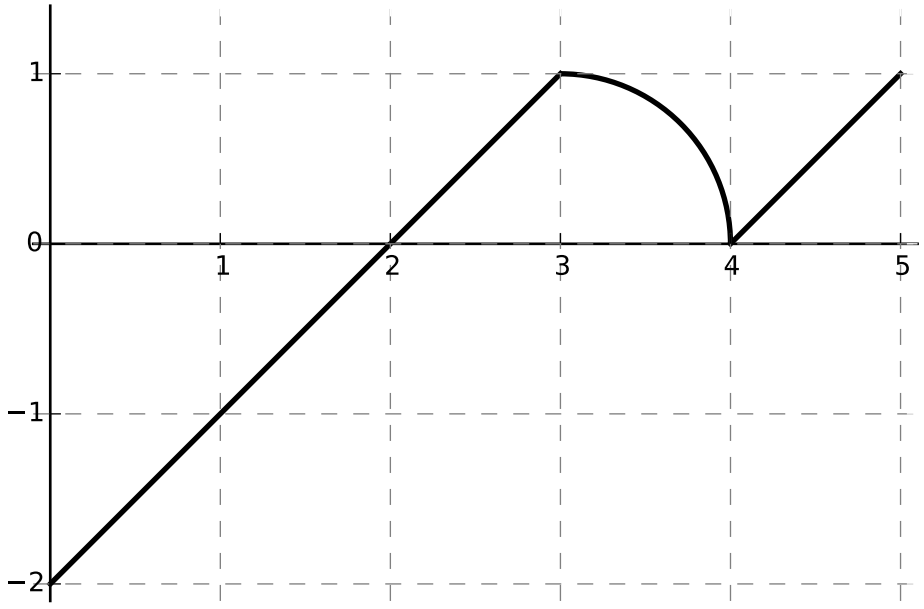

Figure 1: The complete graph of a function