Calc II - Review for exam III

The third exam will be this Friday, November 20. We will discuss some of these problems in class on Wednesday, but you should work them all out to the best of your ability prior to that. Understanding the problems on this sheet will help you greatly on the exam.

1. Suppose we spin the region under the graph of $f(x) = 5x(1-x)$ and over the x-axis around the x-axis. What is the volume of the resulting solid?
2. My one man tent is three feet high and is held up by two pieces of tent cord so that the base is three times as long as it is wide. What is the volume of that tent?
3. Write down an integral representing the arc length of the graph of $y = \sin(x^3)$ over the interval $[0, 2]$.
4. Use the integral test to show that $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n(\ln(n))^p}$ converges precisely when $p > 1$.
5. Classify the following series as absolutely convergent, conditionally convergent, or divergent. Be sure to provide a clear and grammatically correct explanation.
   (a) $\displaystyle\sum (-1)^n \frac{n^2}{n^2+2}$
   (b) $\displaystyle\sum (-1)^n \frac{n^2}{n^3+2}$
   (c) $\displaystyle\sum (-1)^n \frac{n^2}{n^4+2}$
6. Use the comparison test to show that
$$\sum_{n=1}^{\infty} \frac{\sin^2\left(n^3\right)}{n^4}$$
converges.
7. Let $f(x) = \sqrt[3]{x}$. Find the quadratic approximation of f centered at $x_0 = 1$.
8. Use a power series expansion to evaluate the following integrals:
   (a) $\displaystyle\int x^3 e^{x^3} dx$
   (b) $\displaystyle\int \frac{x^2}{1+x^5} dx$
9. Use the geometric series formula to express $0.21\overline{12}$ as a fraction.
10. Starting with a square of side length 1, we replace each side with a 7 segments of length 1/5 the length of the original segment as shown in the figure below on the left. We then repeat this process obtaining in the limit a fractal object as shown on the right. What is the area of this object?

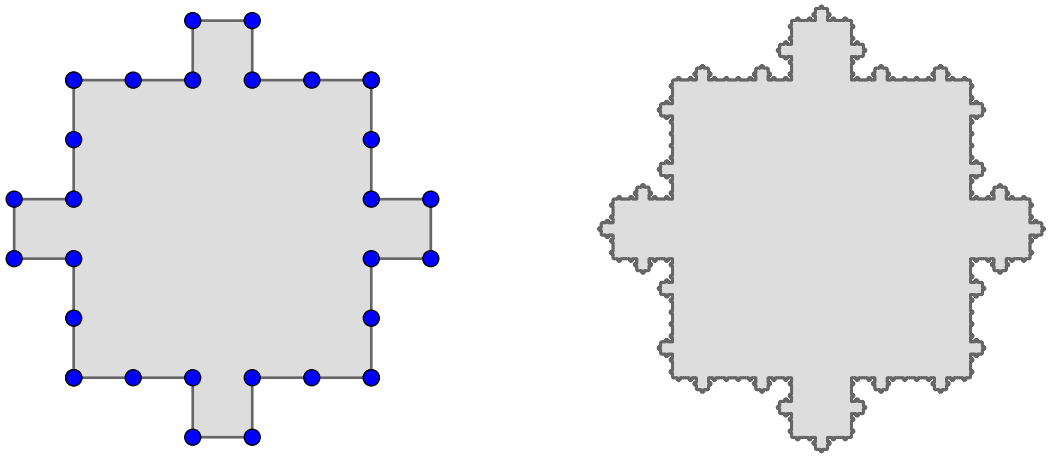

Figure 1: A fractalized square