Problems - HW 6

Tuesday, September 22

- 1. Suppose that V is a vector space of dimension 7, that W is a vector space of dimension 4, and that $T:V\to W$ is a linear transformation. Which of the following is *not* necessarily true?
 - (a) If T(v) = 3w, then T(2v) = 6w.
 - (b) There are independent vectors $v_1, v_2 \in V$ such that $T(v_1) = T(v_2) = 0$.
 - (c) $T(v_1 v_2) = T(v_1) T(v_2)$.
 - (d) For every $w \in W$, there is a $v \in V$ such that T(v) = w.
 - (e) There are independent vectors $v_1, v_2 \in V$ such that $T(v_1) = T(v_2)$.
- 2. If V and W are subspaces of \mathbb{R}^8 with $\dim(V) = 5$ and $\dim(W) = 6$, then which of the following must be true.
 - (a) $\dim(V \cap W) = 1$
 - (b) $\dim(V \cap W) = 5$
 - (c) $3 \leq \dim(V \cap W) \leq 5$
 - (d) $7 \le \dim(V + W) \le 10$
 - (e) $\dim(V + W) = 11$
- 3. Which of the following is *not* a subspace of \mathbb{R}^3 ?
 - (a) $\{(x, y, z) \in \mathbb{R}^3 : x 3y + 2z = 0\}$
 - (b) $\{(x,y,z) \in \mathbb{R}^3 : x^2 y^2 + z^2 = 0\}$
 - (c) $\{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 0\}$
 - (d) The y-axis
 - (e) The xz-plane