Calc II - Review for exam III

The third exam will be next Monday, November 21. We will discuss some of these problems in class on Friday, but you should work them all out to the best of your ability prior to that. Understanding the problems on this sheet will help you greatly on the exam.

1. Use the geometric series formula to express $0.21\overline{12}$ as a fraction.
2. Use the integral test to show that $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n(\ln(n))^p}$ converges precisely when $p > 1$.
3. Suppose we'd like to approximate
$$\sum_{n=1}^{\infty} \frac{n}{n^4+1}$$
by truncating the sum to obtain a finite sum of the form
$$\sum_{n=1}^{N} \frac{n}{n^4+1}.$$
How large does N have to be to ensure that our approximation is within 0.0001 of the actual value?
4. Write down a couple of complete sentences using the comparison test to show that
$$\sum_{n=1}^{\infty} \frac{\sin\left(n^3\right)}{n^4}$$
converges absolutely.
5. Write down a couple complete sentences using the alternating series test to show that
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln(n)}{n}$$
converges conditionally.
6. Use the ratio test to determine whether
$$\sum_{n=1}^{\infty} \frac{n^2}{n!}$$
converges.
7. Classify the following series as absolutely convergent, conditionally convergent, or divergent. Be sure to provide a clear and grammatically correct explanation.
   (a) $\displaystyle\sum (-1)^n \frac{n^2}{n^2+2}$
   (b) $\displaystyle\sum (-1)^n \frac{n^2}{n^3+2}$
   (c) $\displaystyle\sum (-1)^n \frac{n^2}{n^4+2}$
8. Starting with a square of side length 1, we replace each side with a 7 segments of length 1/5 the length of the original segment as shown in the figure below on the left. We then repeat this process obtaining in the limit a fractal object as shown on the right. What is the area of this object?

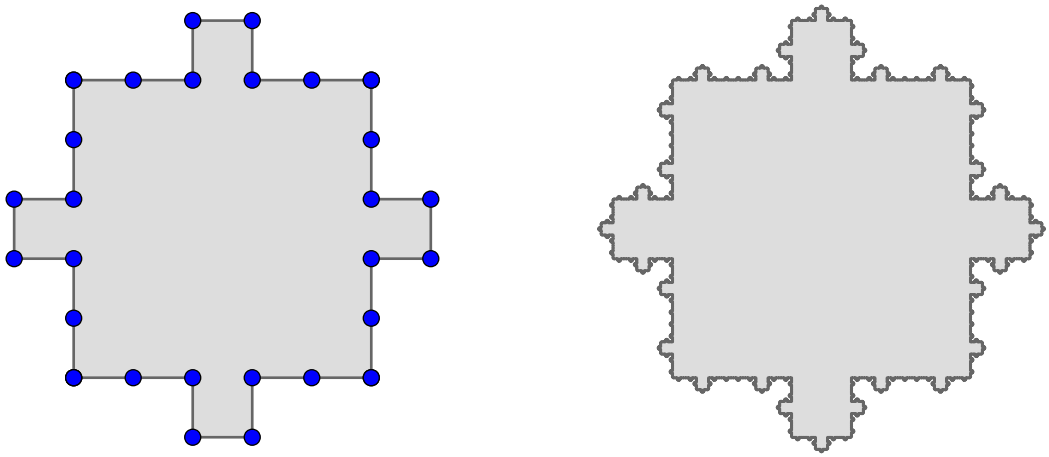

Figure 1: A fractalized square