# Calc I - Final Review

The final exam will be this coming Friday, December 7. This review sheet consists of problems off of our previous three exams together a few problem on the material - namely integration.

## Exam 1

2. Figure 1 shows the complete graph of a function $f$. Evaluate the following or write DNE.

   (a) $f(0)$                                                            Answer:_____

   (b) $f(2)$                                                            Answer:_____

   (c) $\lim\limits_{x \to 0} f(x)$                                              Answer:_____

   (d) $\lim\limits_{x \to 2^-} f(x)$                                           Answer:_____

   (e) $\lim\limits_{x \to 2^+} f(x)$                                           Answer:_____

   (f) $\lim\limits_{x \to 2} f(x)$                                            Answer:_____



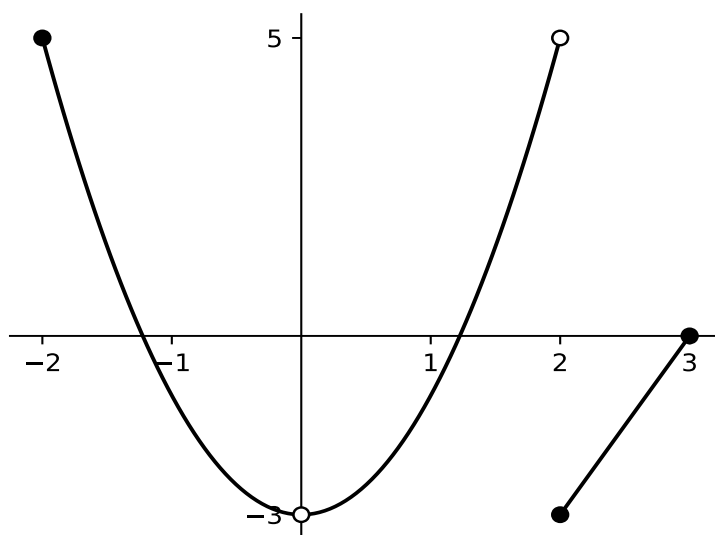Figure 1: The complete graph of a function

3. Compute the following limits.

(a) $\lim\limits_{x \to 2} \dfrac{2x^2 - x - 6}{x - 2}$

(b) $\lim\limits_{x \to 2} \dfrac{2x^2 - x - 4}{x + 2}$

(c) $\lim\limits_{x \to 2^-} \dfrac{2x^2 - x - 4}{x - 2}$

4. Find the derivatives of the following functions, *using the definition of the derivative.*

(a) $f(x) = x^2 - 2x - 1$

(b) $f(x) = \dfrac{1}{x^2}$

5. Find the derivatives of the following functions, using the differentiation rules.

(a) $f(x) = x^2 - 2x - 1$

(b) $f(x) = \dfrac{1}{x^{42}}$

(c) $f(x) = \dfrac{x^4 + 1}{x^2}$

(d) $f(x) = 11^x$

6. Let $f(x) = x^5 - x$. Find an equation of the line that is tangent to the graph of $f$ at the point where $x = 1$.

7. If $f$ and $g$ are differentiable function, prove that

$$\frac{d}{dx}(2f(x) - g(x)) = 2f'(x) - g'(x)$$

8. The complete graph of a function $f$ is shown in figure 2. Sketch the graph of $f'$ on the provided axes.

## Exam 2

1. Evaluate the derivatives of the following functions using the differentiation rules:

(a) $f(x) = e^x + \sin(x) - \cos(x)$

(b) $f(x) = e^x \arcsin(x) - \cos(1/x)$

(c) $f(x) = x^2 \sec(x)(x^3 + x + 1)$

(d) $f(x) = \dfrac{x \tan(x)}{(x^2 + x) \sin(x)}$

(e) $f(x) = \arctan(x^3)$

4. In this problem, we'll use the pre-drawn axes shown in figure **??** to draw the graphs of $f(x) = \cos(2x)$ and $f'(x) = -2\sin(2x)$ and explore their relationship.

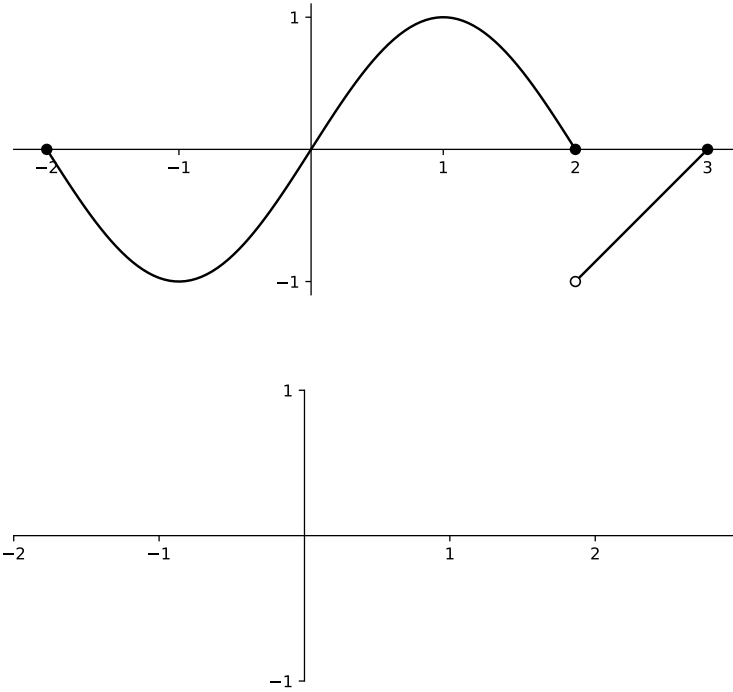(a) Sketch the graph of $f(x) = \cos(2x)$ on the top pair of axes.

Figure 2: The complete graph of a function with a spare set of axes

(b) Sketch the graph of $f'(x) = -2\sin(2x)$ on the bottom pair of axes.

(c) Identify all points with horizontal tangent lines on the top graph. To what points do these correspond in the bottom graph?

5. Use a linear approximation to find a good estimate to $\sqrt[3]{27.6}$.

6. Let $f(x) = x^3 - 5$.

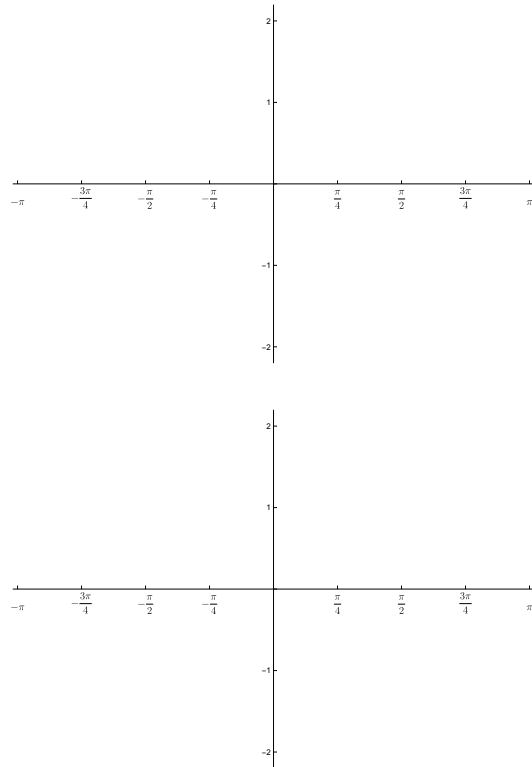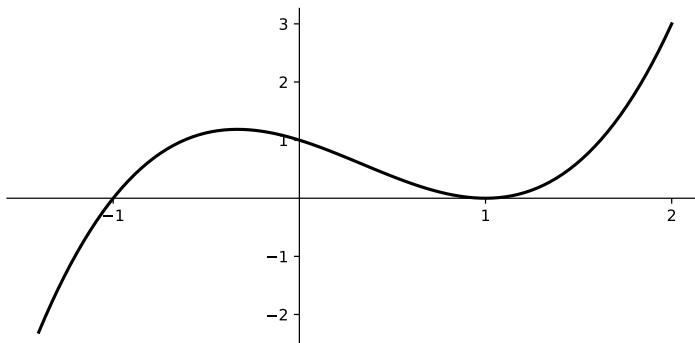(a) Find the corresponding Newton's method iteration function $N(x)$.

Figure 3: A pair of pairs of axes

(b) Perform two Newton iteration steps from the initial point $x_1 = 1$.

7. The figure below shows the graph of $y = x^3 - x^2 - x + 1$.



For that graph:

(a) Identify the local maximum, minimum, and inflection point by marking them on the graph.

(b) Find the exact $x$-coordinate of the local maximum.

(c) Find the exact $x$-coordinate of the inflection point.

# Exam 3

1. Find an equation of the line that is tangent to the curve satisfying $2x^4 - 2x^3y^2 + y^3 = 1$ at the point $(1, 1)$.

2. Let $f(x) = x^3 - 3x^2 + 5$. Find the absolute maximum and minimum values of $f$ over the interval $[0, 4]$.

3. As I drive down a straight highway at 75 mph, a police officer 1.5 miles down the road and 1/4 mile off the road points his radar gun at me. What speed will his radar gun say I'm driving?

4. Suppose I set up a rectangular corral to enclose 4000 square feet with four inner partitions, as shown below. The material for the exterior portion costs three times as much as the material for the interior walls. What are the dimensions of the cheapest such corral?
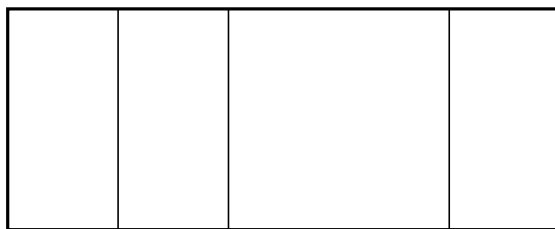


5. Write down a right Riemann with $n = 4$ terms sum to estimate

$$\int_{-1}^{1} \cos(x) \, dx.$$

6. The speed of an object in meters per second is given by $v(t) = 10 - t^2$ over the time interval $0 \leq t \leq 3$.

   (a) Breaking the time interval into 6 pieces and using the speed of the object at $t = 1/2$, $t = 1$, $t = 3/2$, $t = 2$, $t = 5/2$, and $t = 3$, obtain an estimate of the total distance travelled over the time interval.

   (b) Is your estimate a bit bigger or a bit smaller than the actual distance travelled?

7. The speed of another object in meters per second is given by $v(t) = 10 - t$ over the time interval $0 \leq t \leq 3$. Find the exact distance the object travels over the time interval.

# More integration problems

1. Use the Fundamental Theorem of Calculus to evaluate the following definite integrals

   (a) $\displaystyle \int_0^2 (3x^2 + x + 2)\, dx$

   (b) $\displaystyle \int_1^e \left( \frac{1}{x^2} + \frac{1}{x} \right) dx$

2. Use substitution to evaluate the following indefinite integrals

   (a) $\displaystyle \int x^3 \sin(x^4 + 1)\, dx$

   (b) $\displaystyle \int \frac{2x + 1}{x^2 + x + 1}\, dx$

3. In this problem, we'll explore the relationship between a definite integral of the sine function and the area between the graph of the sine and the $x$-axis.

   (a) Sketch the graph of $f(x) = \sin(x)$.

   (b) Evaluate $\displaystyle \int_0^\pi \sin(x)\, dx$.

   (c) Evaluate $\displaystyle \int_0^{2\pi} \sin(x)\, dx$.

   (d) What is the area between the graph of the sine function and the interval $[0, 2\pi]$ on the $x$-axis?

4. The complete graph of a function is shown in the figure below; it consists of two line segments and a quarter circle. Evaluate
$$\int_{-2}^4 f(x)\,dx.$$