Stat 225 - Review for exam I

The first exam will be this Friday, September 28. Here are some problems that might help. Note that you may use a simple calculator.

1. Let $D = \{4, 25, 5, -1\}$.
   (a) Write down a computation showing that the mean of D is 5.
   (b) Express the standard deviation of D as the square root of a sum.
   (c) What is the median of D?
2. Suppose that, amongst the UNCA students majoring in the natural sciences at UNCA,
   - 35% are math majors,
   - 25% are CS majors, and
   - 15% are double majoring in math and CS.

   We then randomly select a UNCA science major and let A denote the event that student is a math major and B denote the event that student is a CS major.

   Write down the following events symbolically in terms of A and B and compute their probability.
   (a) the event that we randomly select a student majoring in math *or* CS.
   (b) the event that we randomly select a student double majoring in math *and* CS.
   (c) the event that we randomly select a math major, given that they are majoring in CS.
3. I've got an eight-sided die with three sides labeled 1, two side labeled 2, and three side labeled 3.
   (a) Show that the expected roll is 2.
   (b) Show that the standard deviation of one roll is $\sqrt{3}/2$.
   (c) What are the expectation and standard deviation of 100 rolls?
4. Let X denote the random variable defined by
   - $P(X = 1) = 0.2$
   - $P(X = 2) = 0.3$
   - $P(X = 4) = 0.5$

Let $\{X_i\}_{i=1}^{100}$ denote a sequence of 100 independent trials of X and let

$$S = \sum_{i=1}^{100} X_i.$$

(a) Write down a computation showing that $E(X) = 2.3$.

(b) Write down a computation showing that $\sigma^2(X) = 1.09$.

(c) Use a normal table to estimate $P(S < 222)$.

5. I've got some cuboctahedral die, each with 8 triangular faces labeled zero and 6 square faces labeled 1. When I roll one of these die,

- The probability I get a triangle is $2\sqrt{3}/(2\sqrt{3}+6) \approx 0.366$ and
- The probability I get a triangle is $6/(2\sqrt{3}+6) \approx 0.634$

Let's let X be the random variable whose value the numeric result of a roll.

(a) Write out the definition of X.

(b) Compute the expectation of X.

(c) Compute the standard deviation of X.

(d) Suppose I roll 10 such die. What is the probability that I get exactly 4 triangles?

(e) Suppose I roll 10 such die. What is the probability that I get at most 4 triangles?

(f) Suppose I roll 100 such die. What is the probability that I get at most 42 triangles?

(g) Suppose I roll 100 such die. What is the probability that I get at most 99 triangles?

6. Asheville has three orthopedic centers that perform wrist surgery: BRBJ, AO, and CH. These places perform 40%, 38%, and 22% of wrist surgeries in the area respectively. Of theses surgeries,

- 0.1% from BRBC result in post surgical complications,
- 0.2% from AO result in post surgical complications, and
- 0.3% from CH result in post surgical complications.

Suppose that a randomly chosen wrist surgery patient had post surgical complications. What is the probability that patient is from AO?

7. The PDF of a continuous random variable X is shown in figure 1.

(a) Explain why the figure represents a good PDF.

(b) Compute $P(2 < X < 3)$.

(c) Compute $P(3 < X < 5)$.

8. Suppose that the PDF of a continuous random variable on $[0, 1]$ is given by

$$f(x) = \begin{cases} cx^2 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}.$$

(a) Find the value of c that makes f a good PDF.

(b) Compute $P(0 < X < 1/2)$

(c) Compute $E(X)$

(d) Compute $\sigma(X)$

9. Suppose that X is normally distributed with mean 555 and standard deviation 28. Find $P(540 < X < 600)$.

10. Suppose my classes exam scores are normally distributed with a mean of 60 and a standard deviation of 15. What percentage of my students score above 90%?

11. Use a u-substitution to translate the normal integral

$$\frac{1}{\sqrt{20\pi}} \int_0^5 e^{-(x-2)^2/20} dx$$

into a *standard* normal integral.

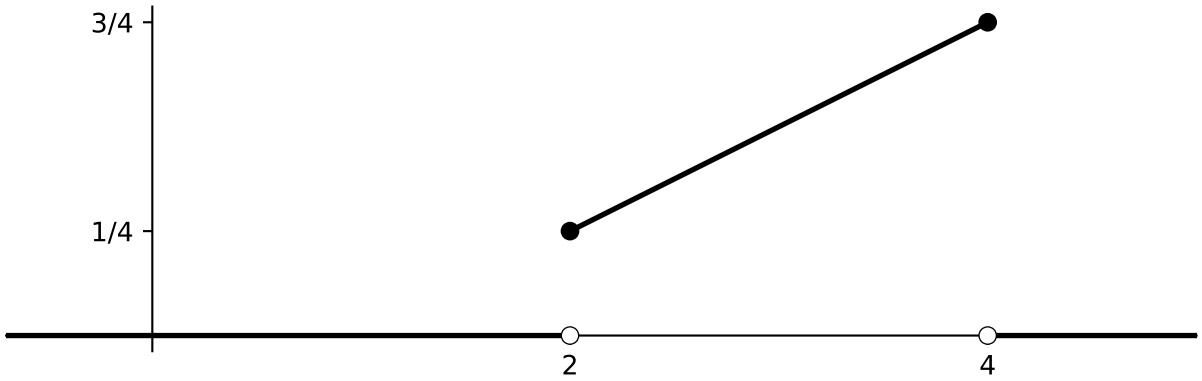

Figure 1: A PDF