Calc I - Final Review

Exam I

2. Use the following steps to *estimate* the derivative of $f(x) = 4^x$.
   (a) Write down the difference quotient for f.
   (b) Use a little algebra to separate the xs from the hs.
   (c) Refer to the following table to help you find your final estimate.

h	0.1	0.01	0.001	0.0001	0.00001	0.000001
$\frac{4^h - 1}{h}$	1.4869835	1.3959480	1.3872557	1.386390	1.386304	1.386295

3. Find the derivatives of the following functions, *using the definition of the derivative.*
   (a) $f(x) = x^3 - x$
   (b) $f(x) = \sqrt{x}$
4. Find the derivatives of the following functions, using any technique you see fit.
   (a) $f(x) = x^3 - x - \sqrt{x}$
   (b) $f(x) = x^2 + x^e + e^x + 2^x$
   (c) $f(x) = \dfrac{x-1}{\sqrt{x}}$
6. The complete graph of a function f is shown in figure 1. Sketch the graph of f'.

Exam II

1. Find the derivatives of the following functions.
   (a) $f(x) = \sin(x) + \cos(x) + \ln(x) + \arcsin(x) + \arctan(x)$
   (b) $f(x) = \sin(x^2)$
   (c) $f(x) = x^2 \arcsin(x)$
   (d) $f(x) = \dfrac{x^2 - x - 1}{\sin^2(x)}$
   (e) $f(x) = x \arctan(x^3)$

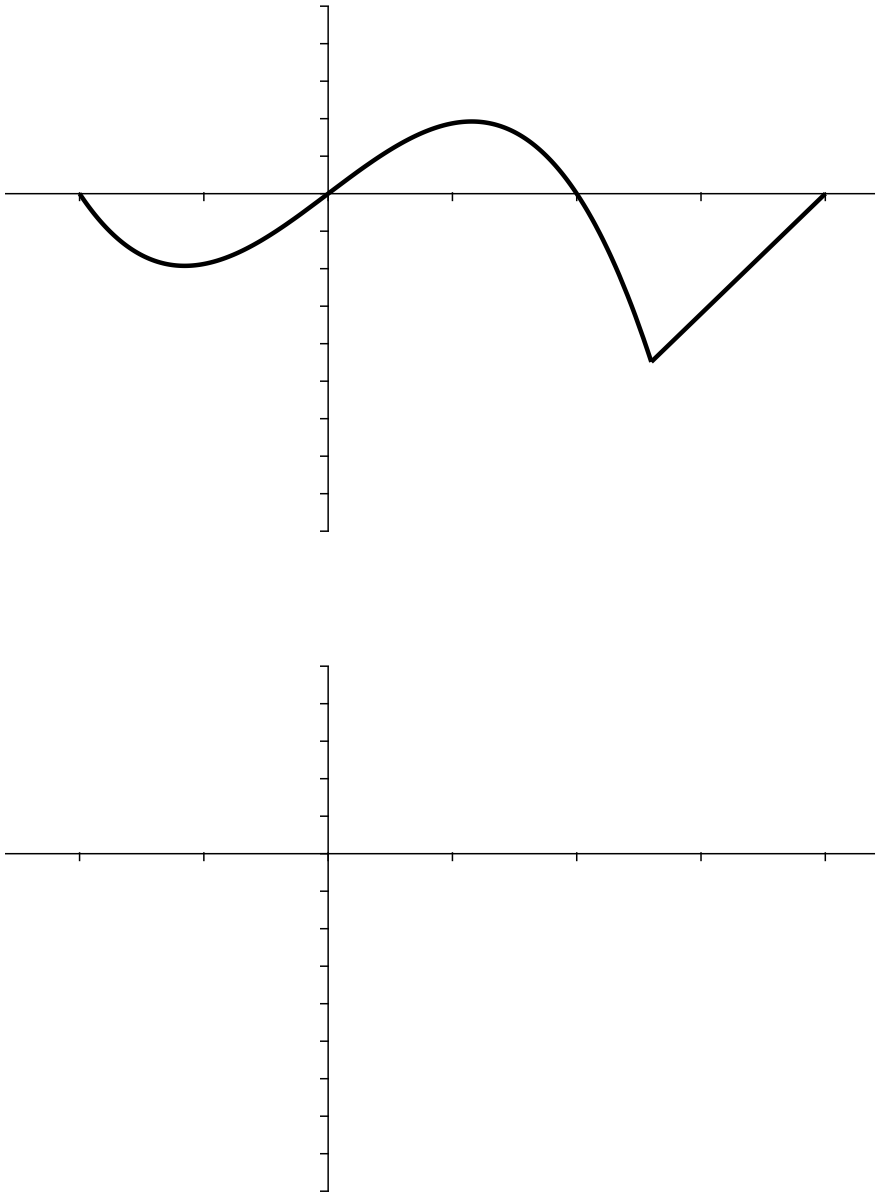

Figure 1: The graph of a function with a spare set of axes

(f) $f(x) = \ln\left(\dfrac{x\sqrt{x+1}}{(x+2)^2}\right)$

5. Let $f(x) = \sin(2x)$. Find an equation for the line tangent to the graph of f at the point $(\pi/8, f(\pi/8)$.

7. In this problem, you're going to use the definition of the derivative to show that

$$\frac{d}{dx} x\, f(x) = f(x) + x\, f'(x)$$

using the following outline:

- Write down the difference quotient for the expression $x\, f(x)$.
- Try to rewrite your expression so that you can identify the parts you need to get the desired limit.
- Take that limit!!

Exam III

1. Let $f(x) = x^2 e^x$. A graph of f over the intervale $[-10, 0]$ is shown in figure 2.
   (a) Find the exact locations of the absolute maximum and minimum of f over $(-\infty, 0])$.
   (b) Find the exact locations of the inflection points of f over that same interval,

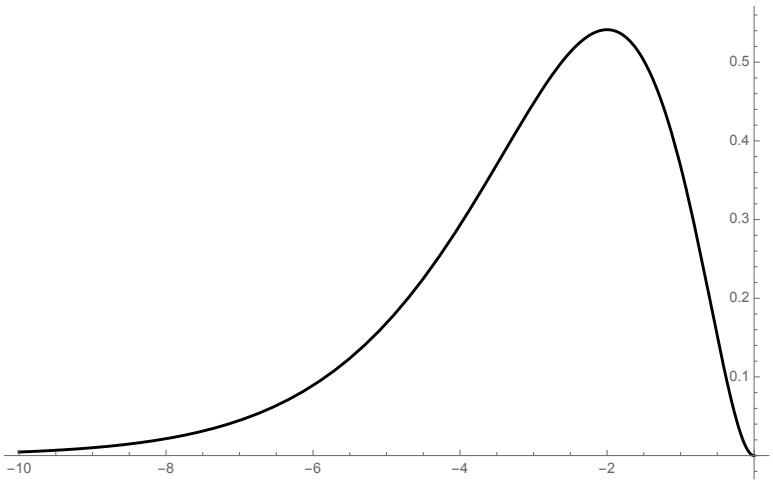

Figure 2: The graph of $f(x) = x^2 e^x$

2. The police are after me again, as shown in figure 3. Suppose I'm 2 miles from the intersection and driving away from it at 50 mph while the the kindly police people are 1 mile away from the intersection and driving towards it at 60 mph. At what rate is the distance between the the police and me changing? Be sure to carefully account for the sign.

3. Suppose I have 400 ft of fence to enclose a corall against an existing wall (so that I only need to suse the fence for three sides of the corall). What is the largest possible area I can enclose and how should I set up the fence to obtain that area?

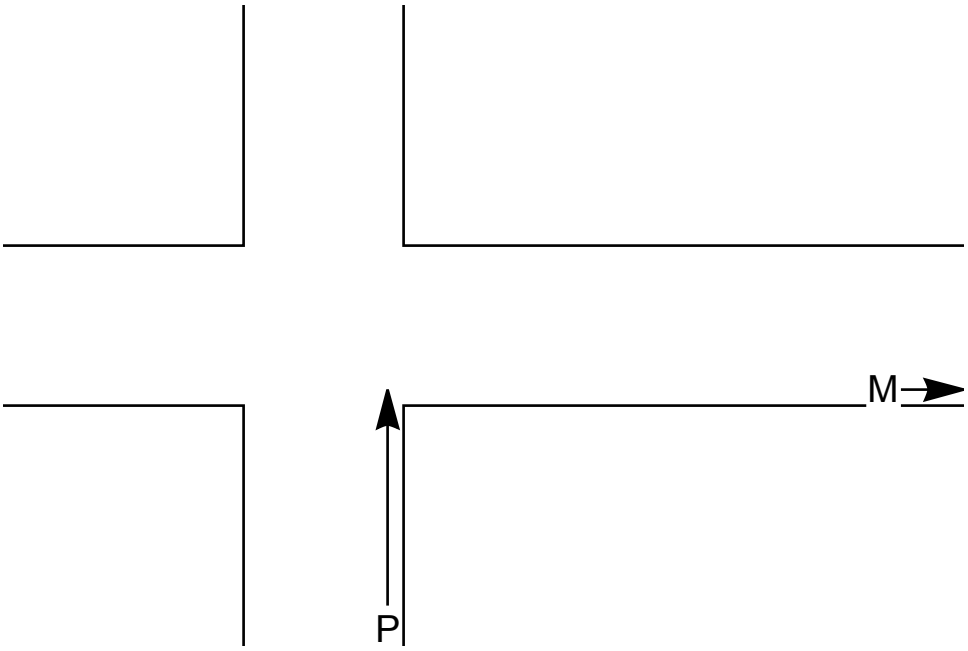

Figure 3: Chasing a bad person

u-Substitution

1. Use u-substitution to evaluate the following indefinite integrals.
   (a) $\int x\sqrt{x^2+1}\,dx$
   (b) $\int \sqrt{2x+1}\,dx$
   (c) $\int \sin^3(x)\cos(x)\,dx$
   (d) $\int \dfrac{1}{x\ln(x)}\,dx$
2. Use u-substitution to evaluate the following definite integrals.
   (a) $\displaystyle\int_1^2 x^2(x^3+1)^9\,dx$
   (b) $\displaystyle\int_{-1}^1 xe^{\sin(x^2)}\,dx$
3. Use u-substitution to express the following normal integral as a *standard* normal integral:

$$\frac{1}{\sqrt{2\pi}4}\int_{-1}^2 e^{-(x-1)^2/32}\,dx.$$