Calc I - Review for exam I

The first exam will be next Wednesday, September 15. Here are some problems that might help.

Table 1: Usain Bolt's splits for the the 2008 Olympic 100m race

0-10m	10-20m	20-30m	30-40m	40-50m	50-60m	60-0m	70-80m	80-90m	90-100m	Total
1.85	1.02	0.91	0.87	0.85	0.82	0.82	0.82	0.83	0.90	9.69

1. Table 1 shows the splits for Usain Bolt's 100m race at the 2008 Olympics.
   (a) What was his average speed for the race in meters per second?
   (b) What is your best estimate his top speed?
2. Use the following steps to *estimate* the derivative of $f(x) = 6^x$.
   (a) Write down the difference quotient for f.
   (b) Use a little algebra to separate the xs from the hs.
   (c) Refer to the following table to help you find your final estimate.

t	0.1	0.01	0.001	0.0001	0.00001	0.000001
$\frac{6^h - 1}{h}$	1.96231	1.80791	1.79337	1.79192	1.79178	1.79176

3. Let $f(x) = x^2 - x$.
   (a) Draw the graph of f.
   (b) Write down the difference quotient for f and simplify it to the point where you can cancel the h in the denominator.
   (c) Write down the estimate for the slope of the graph of f at $x = 1$ yielded by the difference quotient using $h = 0.2$.
   (d) Write down an equation for the corresponding secant line through $(1, f(1))$ and $(1.2, f(1.2))$ and draw it on your graph.
   (e) Write down and equation for the precise tangent line at $x = 1$ and draw that on your graph as well.
4. Figure 1 shows the complete graph of a function f; its domain is $(-1, 2]$.
   (a) What are $f(1.5)$ and $\lim_{x\to 1.5} f(x)$?
   (b) What are $\lim_{x\to 1^-} f(x)$, $\lim_{x\to 1^+} f(x)$, and $\lim_{x\to 1} f(x)$?
   (c) What can you say about $\lim_{x\to -1^+} f(x)$.

5. Find the derivatives of the following functions, *using the definition of the derivative.*
   (a) $f(x) = 2x^2 - 4x$
   (b) $f(x) = 1/\sqrt{x}$
   (c) $f(x) = x^5$
6. Find the derivatives of the following functions, using any technique you see fit.
   (a) $f(x) = 2x^2 - 4x$
   (b) $f(x) = 1/\sqrt{x}$
   (c) $f(x) = x^5$
   (d) $f(x) = x^5\left(x^2 - x - 1\right)$
   (e) $f(x) = \left(x^2 - x - 1\right)/x$
   (f) $f(x) = 2^x + 7^x + e^x + x^e$
7. Find the first and second derivatives of $f(x) = \sqrt{x}$.
8. The complete graph of a function f is shown in figure 2; it consists of a line segment, a quarter-circle, and a semi-circle. Sketch the graph of f'.
9. The complete graph of a function f is shown in figure 3.
   (a) On what intervals is $f' > 0$?
   (b) On what intervals is $f'' > 0$?
   (c) At what points is $f' = 0$?
   (d) At what points is $f'' = 0$?
   (e) At what points is f discontinuous and why?
   (f) At what points is f not differentiable and why?
10. If f and g are differentiable function, prove that

$$\frac{d}{dx}(2f(x) + 3g(x)) = 2f'(x) + 3g'(x)$$

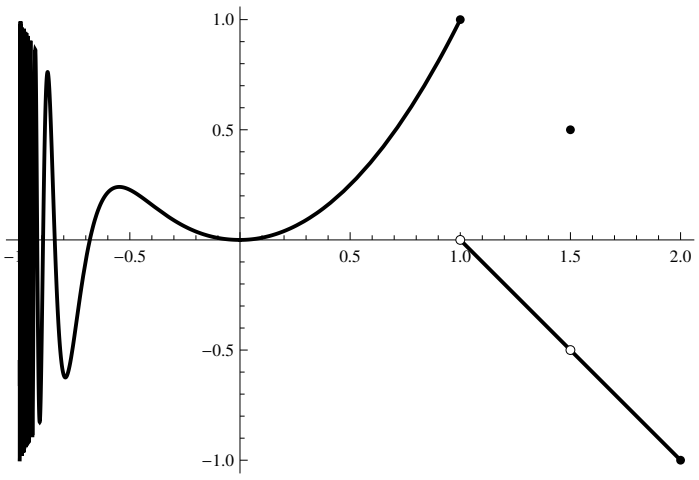

Figure 1: The graph for problem 4

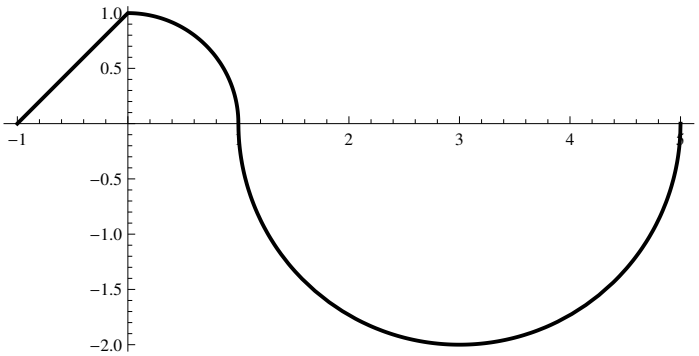

Figure 2: The graph for problem 8

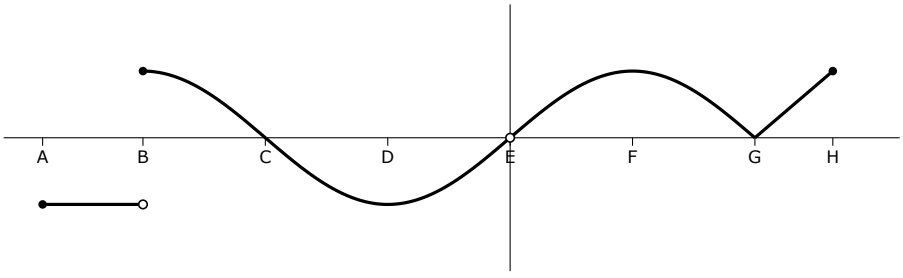

Figure 3: The graph for problem 9