Calc I - Review for exam II

The next midterm exam will be next Wednesday, October 13. Here are some problems that might help.

1. Find the derivatives of the following functions.
   (a) $f(x) = \tan(x) + \sec(x) + \ln(x) + \arcsin(x) + \arctan(x)$
   (b) $f(x) = x^2 + \dfrac{1}{x}$
   (c) $f(x) = e^x + \dfrac{1}{x}$
   (d) $f(x) = \dfrac{x^5}{\sin(x)}$
   (e) $f(x) = x^7\left(x^2 + x\right)$
   (f) $f(x) = \sqrt{\sin(x)} + \cos(x)$
   (g) $f(x) = \dfrac{1}{x^7}\cos\left(x^7\right)$
   (h) $f(x) = x^4\cos^2(x)$
   (i) $f(x) = \dfrac{e^x}{x^3}\left(x + \dfrac{1}{x}\right)$
   (j) $f(x) = \dfrac{4^{-x}}{x^4}$
   (k) $f(x) = \dfrac{e^{-x}\cos(x)}{\sin(\cos(x))}$
   (l) $f(x) = \tan(x^3) + x^3\sec(x)$
   (m) $f(x) = \arctan(x^3) + x^3\arcsin(x)$
   (n) $f(x) = x\ln(3x)$
   (o) $f(x) = \tan(2x)\arctan(x^2)$
2. Use the formula for the derivatives of the sine and cosine together with the combination rules for differentiation to establish the following formulae.
   (a) $\dfrac{d}{dx}\tan(x) = \sec^2(x)$
   (b) $\dfrac{d}{dx}\sec(x) = \sec(x)\tan(x)$.

3. In this problem, we're going to derive the fact that, if $f(x) = \ln(2x)$, then $f'(x) = 1/x$ using the fact that we know the inverse of f.
   (a) Starting from $y = \ln(2x)$, write the equation in exponential form.
   (b) Implicitly differentiate your equation from part (a) with respect to x.
   (c) Solve your equation from part (b) for y'.
   (d) Simplify, if necessary to show that $y' = 1/x$.
4. Compute the derivative of the arccosine using the following recipe.
   (a) Write $y = \arccos(x)$, apply the cosine to both sides, and simplify.
   (b) Apply implicit differentiation to your response to part (a) and solve for y' in terms of y.
   (c) Express your solution to the previous part in term of x.
5. Sketch the graphs of each of the following functions.
   (a) $f(x) = \sin(2x - \pi)$
   (b) $f(x) = 2\cos(\pi x) + 2$
6. In this problem, we'll use the pre-drawn axes in figure 1 to draw the graphs of $f(x) = \sin(2x)$ and $f'(x) = 2\cos(2x)$ and explore their relationship.
   (a) Sketch the graph of $f(x) = \sin(2x)$ on the top pair of axes.
   (b) Sketch the graph of $f'(x) = 2\cos(2x)$ on the bottom pair of axes.
   (c) Identify all points with horizontal tangent lines on the top graph. Projecting down, does the bottom graph cross the x axis at those points?
7. Let $f(x) = x\sin(2x)$. Find an equation for the line tangent to the graph of f at the point $(\pi/4, f(\pi/4)$.
8. Figure 2 shows the graph of $f(x) = x^3 + \frac{1}{2}x - \frac{3}{2}$ together with the dashed line $y = x$.
   (a) Sketch the graph of f^{-1} on the same set of axes.
   (b) Find an equation for the line that is tangent to the graph of f^{-1} at the point where $x = 0$.

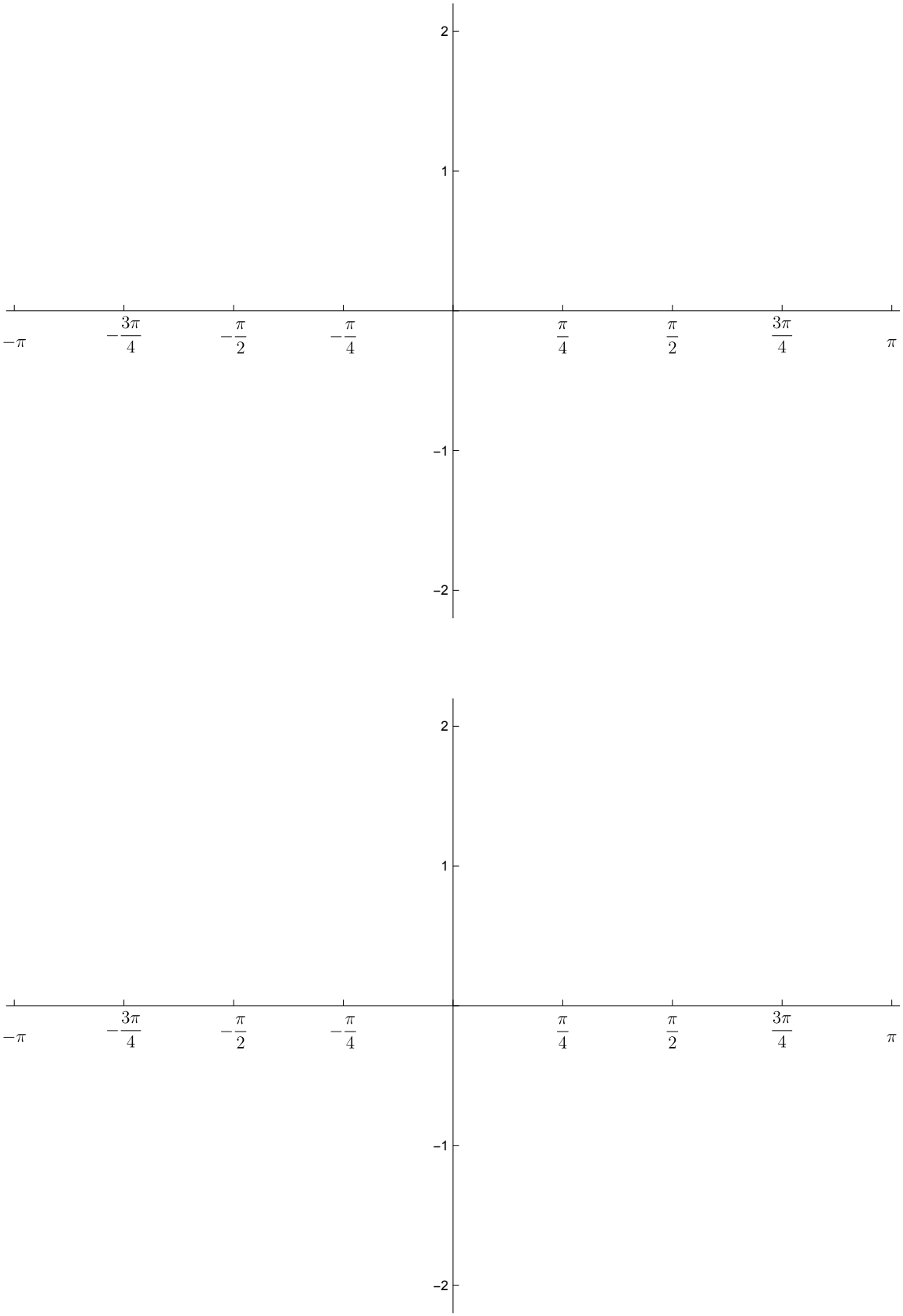

Figure 1: A pair of axes for trig functions

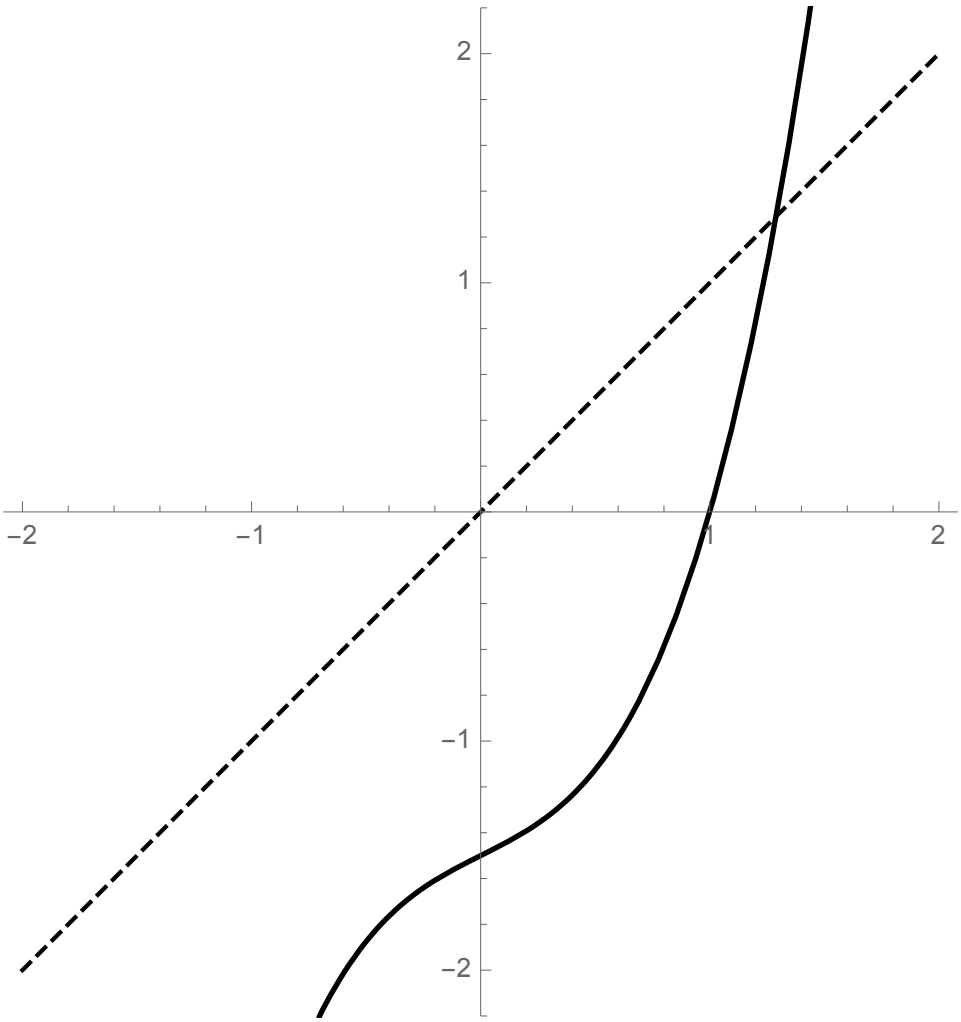

Figure 2: A pair of axes for trig functions