Prep for Quiz 1

text

Our first quiz will be next Wednesday, September 1. There will *certainly* be problems very much like 1 and 2 on the quiz; the probability is 100%. It's very likely there will be something very like 3 and/or 4; maybe a little different. Maybe there will be something else.

1. **Definitions** There will certainly be some version of this problem. You should be able to write down the following definitions (which are all in our text) verbatim - *don't feel the need to put these in your own words*! You can generally assume that we are working with a function $f : \mathbb{R} \to \mathbb{R}$.
   a. *Orbit* of a point x_0 under iteration of f.
   b. *Fixed point* of f.

   c. *Periodic point* and *periodic orbit* of f.
   d. *Attractive*, *super-attractive*, *repelling*, and *neutral* fixed points of f.
   e. *Attractive*, *super-attractive*, *repelling*, and *neutral* peridioc orbits of f.
2. **Graphical Analysis** Refer to figure 1 on the next page.
   a. Identify the fixed points on the figure and classify them as attractive or repulsive.
   b. Perform graphical analysis starting from the green point. What is the fate of the orbit.
3. **Affine Iteration** Suppose that f has the form

$$f(x) = ax + b = a\left(x - \frac{b}{1-a}\right) + \frac{b}{1-a}$$

   where $a \neq 1$.
   a. Find the fixed point of f.
   b. Find a closed form expression for $f^n(x)$.
   c. Specializing to the case where $a = 2$ and $b = -3$ so that $f(x) = 2x - 3$, find a simple expression for $f^{10}(4)$.
   d. Use your closed form expression to explain why any orbit converges to the fixed point if $|a| < 1$.
4. **Variability of neutral orbits** Your mission in this problem is to find 4 functions that all have the origin as a neutral fixed point but with four different behaviors in a neighborhood of that fixed point. Specifically, find four functions f_1, f_2, f_3, and f_4, such that for each $i = 1, 2, 3, 4$, we have $f_i(0) = 0$, $f_i'(0) = 1$, and
   a. Points close to 0 move towards 0 under iteration of f_1,
   b. Points close to 0 move away from 0 under iteration of f_2,
   c. Negative points close to 0 move towards 0 under iteration of f_3 but positive points close to 0 move away from 0 under iteration of f_3
   d. Positive points close to 0 move towards 0 under iteration of f_4 but negative points close to 0 move away from 0 under iteration of f_4.

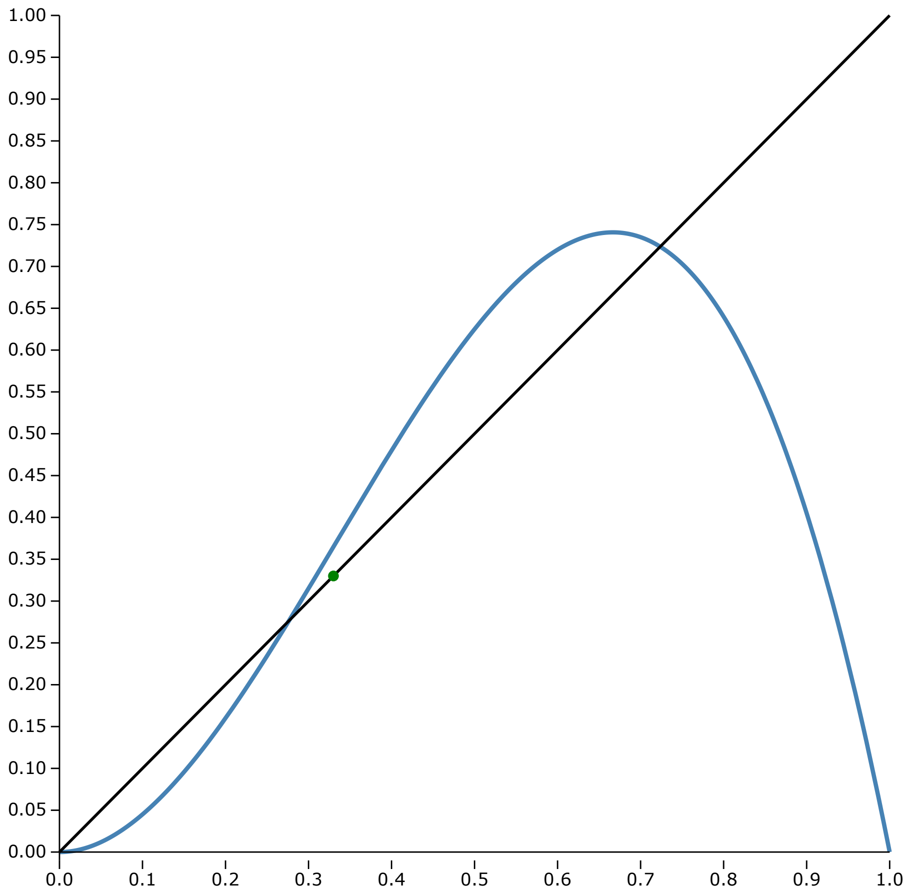

Figure 1: The graph for problem 2