Chaos & Fractals - Writing assignment 1

This 30 point assignment is due to me by email next Thursday, October 14 by midnight. Ideally, you should type it in LaTeX using Overleaf and share your project with me.

In this assignment, you'll explore the relationship between two maps restricted to the Cantor set. One, the *tripling map* is analogous to the doubling map and is fairly easily seen to be chaotic. The other, the *tall tent map*, forms a semi-conjugacy between itself and the tripling map. You'll prove this and use the semi-conjugacy to transfer some intersting results from one to the other.

To be precise, let $t : [0, 1) \to [0, 1)$ denote the tripling map

$$t(x) = 3x \mod 1 = \begin{cases} 3x & 0 \le x \le 1/3 \\ 3x - 1 & 1/3 \le x \le 2/3 \\ 3x - 2 & 2/3 \le x < 1. \end{cases}$$

and let $T : [0, 1] \to [0, 1]$ denote the tall tent map

$$T(x) = \begin{cases} 3x & 0 \le x \le 1/2 \\ 3(1 - x) & 1/2 \le x \le 1. \end{cases}$$

1. Show that the tripling map preserves C, i.e. $t : C \to C$.

2. Find a point of period three in C for the tripling map.

3. Show that the tall tent map also preserves C, i.e. $T : C \to C$.

4. Show T is itself a conjugacy between the two, i.e.

$$T \circ T = T \circ t,$$

   when both are restricted to C.

5. Find a point of period three in C for the tall tent map.

Note: This problem mirrors our study of the doubling map and the tent map (as described in the Tent Map section of our text) but applied to the Cantor set (as described in the Cantor set section of our text).