Calc III - Final Review

Exam I

1. Sketch the curve $9x^2 + z^2 = 3$ in the xz-plane. What does the graph of this equation look like in xyz-space?

4. Find the equation of a plane containing the points

$$(3, 2, 3), (0, 1, 1), \text{ and } (0, 0, 3)$$

   or explain why no such plane exists.

5. Let $p(t) = (1 + t, 2 + t, 1 + t)$ and $q(t) = (7 + 3t, 6 + 2t, -1 - t)$ be the parameterizations of two the paths of two objects moving through space.

   (a) Do the paths intersect?

   (b) Do the motions parameterized by these functions collide?

7. Let $f(x, y) = x^2 - xy + y^3$. Find an equation of the plane tangent to the graph of f at the point $(1, 1)$.

Exam 2

3. Find and classify the critical points of the function $f(x, y) = x^3 - xy + y^2 - y$.

4. Find the equation of the plane tangent to the graph of $x^2 - 2y^2 - 3z^2 = -2$ at the point $(3, 2, 1)$.

5. Let $f(x, y) = 3x + 5y$. In this problem, we're going to find the extremes of f subject to the constraint $x^2 + 9y^2 = 9$.

   (a) Draw the constraint curve together with several contours of f. Use your diagram to identify any maxima or minima of f that you see along the curve.

   (b) Use Lagrange multipliers to find the exact locations of those maxima and minima.

6. Figure 1 shows the contour diagram of

$$f(x, y) = (x + y)e^{-2\left(x^2+y^2\right)}.$$

   Find the exact location of the maxima and minima.

Exam 3

1. Evaluate $\int_0^1 \int_0^2 12x^3y^2 \, dx \, dy$.

3. Set up the double integral
$$\iint_D xy \, dA$$
as an iterated integral where D is the domain stuck between the graphs of $y = x^2 - 1$ and $y = x + 1$.

4. Let D denote the solid pyramid with vertices located at $(1,0,0)$, $(0,1,0)$, $(0,0,2)$, and the origin. Set up an iterated integral to represent the volume of D.

5. Find the volume under the graph of
$$f(x,y) = e^{-(x^2+y^2)}$$
and over the top half of the disk of radius two centered at the origin.

7. Let R denote the region between $f(x,y) = 4 - (x^2 + y^2)$. Set up an iterated integral in cylindrical coordinates representing
$$\iiint_R (x^2 + y^2 + z) \, dV.$$

8. Let D denote the three-dimensional domain inside the sphere of radius one centered at the orgin and also in the first octant (i.e., $x > 0, y > 0, z > 0$). Set up an iterated integral in spherical coordinates representing
$$\iiint_D (x^2 + y^2 + z^2) \, dV.$$

A bit more

1. Let
$$\vec{F}(x,y) = \langle 2y, -x \rangle$$
and let C be the curve parameterized by
$$\vec{r}(t) = \langle t^2, t^3 \rangle$$
over the time interval $-1 \leq t \leq 1$. Compute
$$\int_C \vec{F} \cdot d\vec{r}.$$

2. Let $\vec{F}$ denote the conservative vector field
$$\vec{F}(x,y) = \left\langle 2xy^3 + 1, 3x^2y^2 + 1 \right\rangle.$$
Find a potential function f for $\vec{F}$ and use it to compute
$$\int_C \vec{F} \cdot d\vec{r},$$
where C is a path from the origin to the point $(1,1)$.

3. Let

$$\vec{F}(x, y) = \langle x^2, y^2 \rangle.$$

Use the divergence theorem to compute

$$\int_C \vec{F} \cdot d\vec{n},$$

where C is the boundary of positively oriented unit square.

4. Match the groovy function below with the groovy graph shown in figure 2.

   (a) $\vec{p}(t) = \langle 2\cos(t), \sin(t) \rangle$

   (b) $\vec{p}(t) = \langle 2t\cos(t), t\sin(t) \rangle$

   (c) $\vec{p}(t) = \langle 2\cos(t), \sin(t), t/4 \rangle$

   (d) $f(x, y) = 1 - (4x^2 + y^2)$

   (e) $f(x, y) = e^{-(4x^2+y^2)}$

   (f) $x^2 + 4y^2 + 4z^2 = 4$

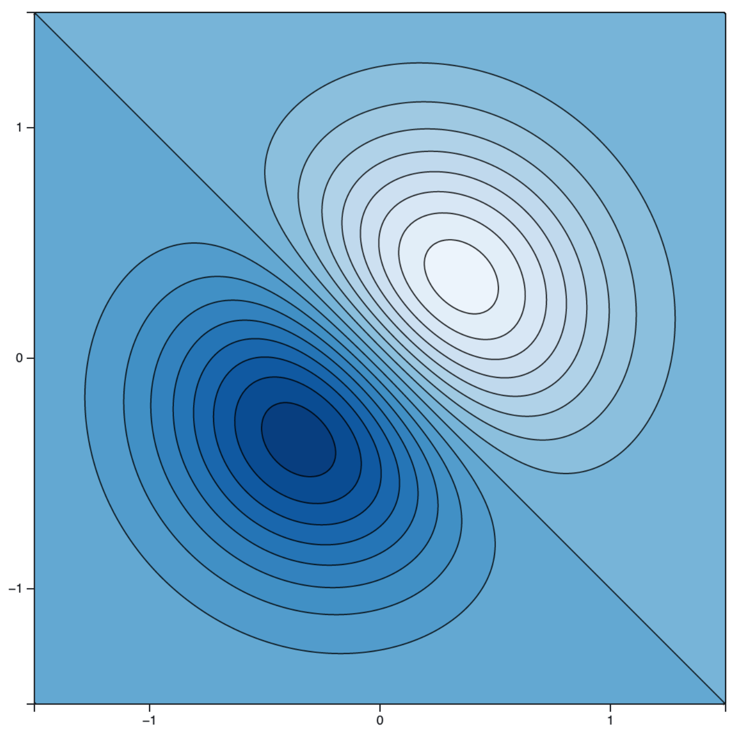

Figure 1: The contour diagram of $f(x, y) = (x + y)e^{-2\left(x^2+y^2\right)}$

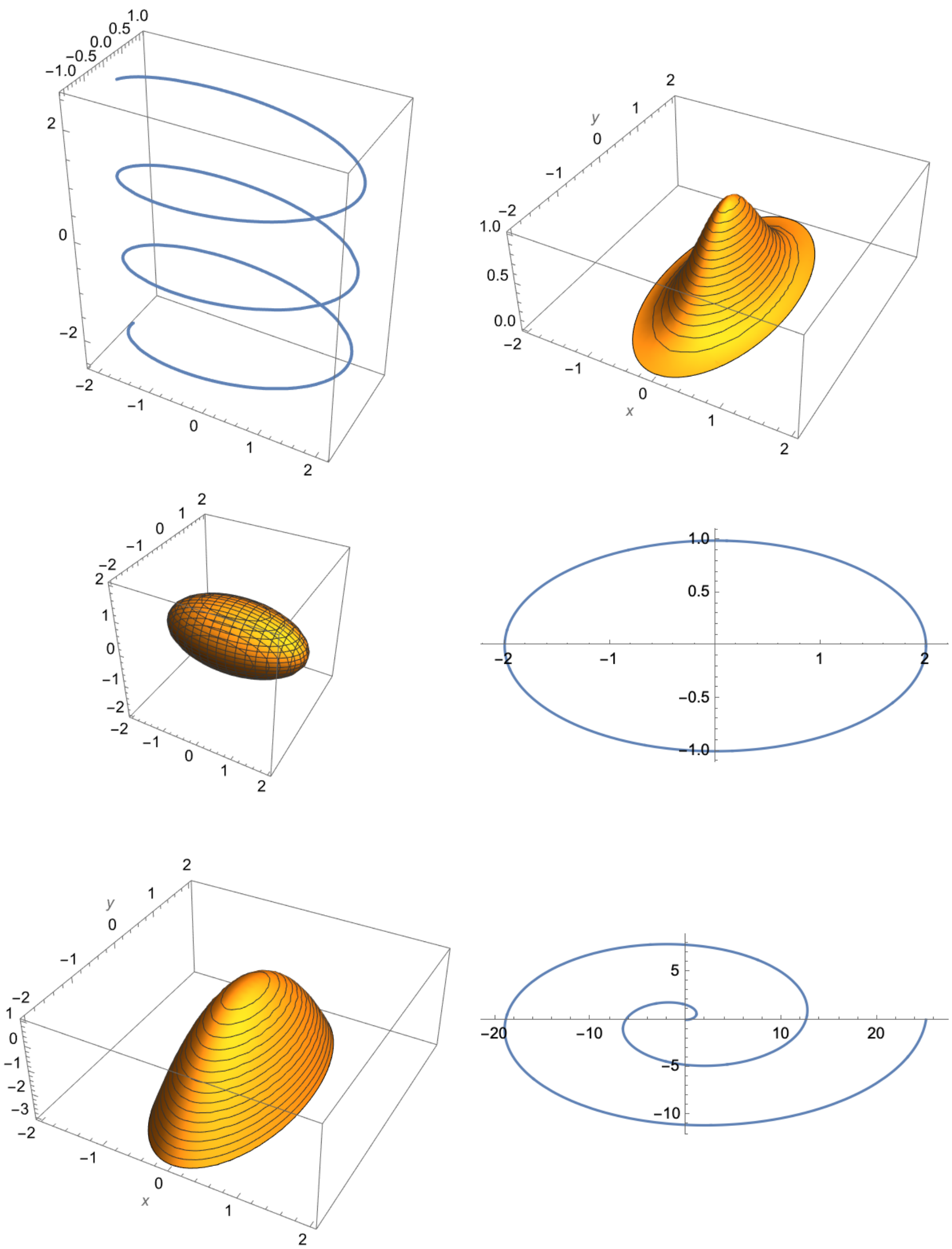

Figure 2: Some groovy graphs