Calc III - Review 3

Exam 3 will be this Friday, November 11. We will go over this problem sheet in class together on Wednesday but *please* be sure work it out to the best of your ability before hand.

1. Evaluate the following double integrals.
   (a) $\int_0^2 \int_0^1 6x^2 y \, dx \, dy$
   (b) $\iint_D x^2 \, dA$, where D is the region in the plane bound between $y = x^2$ and $y = 4$.
   (c) $\int_0^1 \int_y^1 \sin\left(x^2\right) dx dy$
2. Let D denote the solid pyramid with vertices located at $(5, 0, 0)$, $(0, 3, 0)$, $(0, 0, 2)$, and the origin. Set up an iterated integral to represent the volume of D.
3. Find the volume trapped under the graph of the function $f(x, y) = 9 - \left(x^2 + y^2\right)$ and over the xy-plane.
4. Find the volume trapped under the graph of the function $f(x, y) = 9 - \left(x^2 + 9y^2\right)$ and over the xy-plane.
5. Let R denote the region between $f(x, y) = 9 - (x^2 + y^2)$. Evaluate
$$\iiint_R (x^2 + y^2) z \, dV.$$
6. Let R denote the top half of a sphere of radius 2. Set up the triple integral of the arbitrary function f in spherical coordinates.
7. Let D denote the three-dimensional domain above the cone $z = \sqrt{x^2 + y^2}$ and inside the sphere $x^2 + y^2 + z^2 \leq 4$. Evaluate
$$\iiint_D (x^2 + y^2 + z^2) \, dV.$$
8. Find the volume under the surface $f(x, y) = \cos\left(x^2 + y^2\right) + 1$ and over the disk $x^2 + y^2 \leq 3\pi$.
9. Let P denote the parallelogram
$$\{(x, y) : -1 \leq 2x - y \leq 2, \ -2 \leq x + y \leq 1\}.$$
   Set up an iterated integral representing
$$\iint_P xy \, dA.$$

10. Let R denote the region

$$\{(x, y) : 1 \leq x^2 - y^2 \leq 9,\ 2 \leq xy \leq 4\}.$$

Evaluate

$$\iint_R (x^2 + y^2)\, dA.$$

Hint: $\dfrac{\partial(x, y)}{\partial(u, v)} = 1/\dfrac{\partial(u, v)}{\partial(x, y)}$.

11. The graph of $f(x, y) = \sin(xy)$ is shown in figure 1.

(a) Evaluate $\displaystyle\int_{-2}^{2}\int_{-2}^{2} \sin(xy)dxdy$.

(b) Is $\displaystyle\int_{0}^{1}\int_{0}^{1} \sin(xy)dxdy$ positive or negative?

12. A labeled contour plot for a function f is shown in figure 2. Estimate

$$\int_0^1 \int_0^3 f(x, y)\, dx\, dy.$$

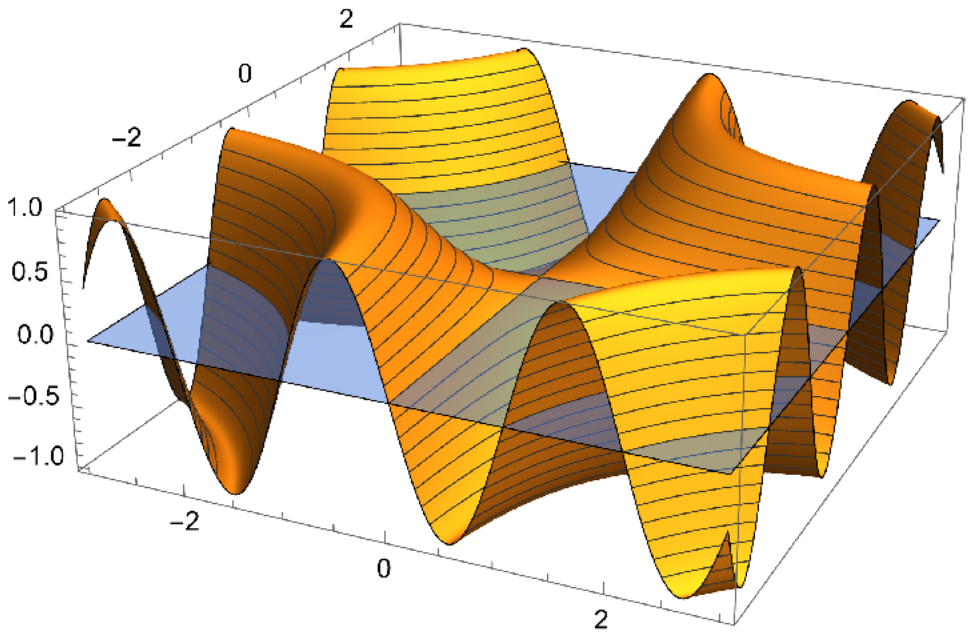

Figure 1: The graph of $f(x, y) = \sin(xy)$

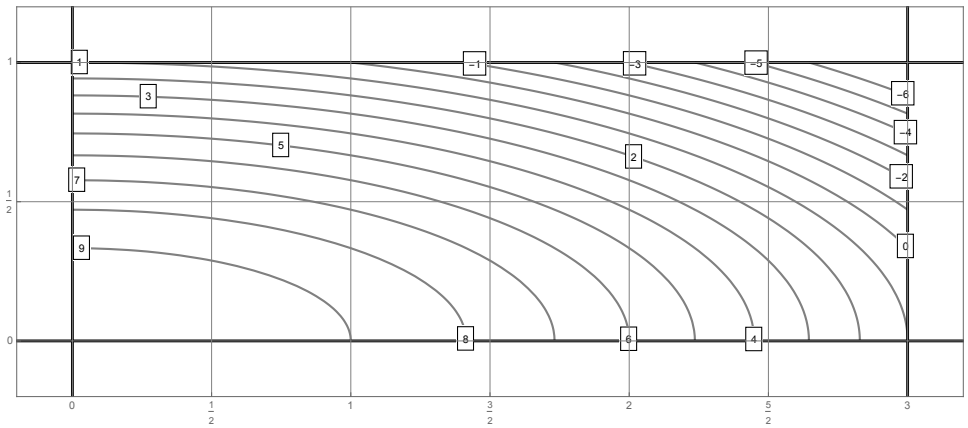

Figure 2: A labeled contour plot