Calc I - Review for exam I

The first exam will be this Friday, February 5. Here are some problems that might help.

Table 1: Values of $(7^h - 1)/h$ vs h

h	0.1	0.01	0.001	0.0001
$(7^h - 1)/h$	2.1481	1.9650	1.9478	1.9461

1. Table 1 shows some values of $(7^h - 1)/h$ versus h. Use the table to estimate

$$\lim_{h \to 0} \frac{7^h - 1}{h}.$$

2. Write down the difference quotient for $f(x) = 7^x$. Use the previous problem to estimate $f'(x)$.

3. Let $f(x) = x^2 + 2x - 1$.

   (a) Sketch the graph of f, together with the line tangent to the graph at the point $(0, f(0))$.

   (b) Compute the slope of the secant line through the points $(0, f(0))$ and $(2, f(2))$. Draw this line on your graph from part (a).

   (c) Write down the difference quotient for f at 0, i.e. the slope of the secant line through the points $(0, f(0))$ and $(0 + h, f(0 + h))$.

   (d) Compute the limit as $h \to 0$ of your answer to part (c).

   (e) How does your answer to part (d) relate to your sketch from part (a)?

4. Let
$$f(x) = \frac{(5x - 1)(x - 3)}{x^2 - 9}.$$

   Evaluate each of the following the limits.

   (a) $\lim_{x \to -1} f(x)$

   (b) $\lim_{x \to -3^-} f(x)$, $\lim_{x \to -3^+} f(x)$, and $\lim_{x \to -3} f(x)$

   (c) $\lim_{x \to 3} f(x)$

5. Figure 1 shows the complete graph of a function f; its domain is $(-1, 2]$.

   (a) What are $f(1.5)$ and $\lim_{x \to 1.5} f(x)$?

   (b) What are $\lim_{x \to 1^-} f(x)$, $\lim_{x \to 1^+} f(x)$, and $\lim_{x \to 1} f(x)$?

   (c) What can you say about $\lim_{x \to -1^+} f(x)$.

6. Find the derivatives of the following functions, *using the definition of the derivative.*

   (a) $f(x) = 2x^2 - 4x$

   (b) $f(x) = 1/\sqrt{x}$

   (c) $f(x) = x^5$

   (d) $f(x) = 1/x^3$

7. Find the derivatives of the following functions, using any technique you see fit.

   (a) $f(x) = 2x^2 - 4x$

   (b) $f(x) = 1/\sqrt{x}$

   (c) $f(x) = x^5$

   (d) $f(x) = x^5 \left(x^2 - x - 1\right)$

   (e) $f(x) = \left(x^2 - x - 1\right)/x$

   (f) $f(x) = e^x \left(x^2 - x - 1\right)$

8. Let $f(x) = x^2 - x$.

   (a) Write down the estimate for the slope of the graph of f at $x = 2$ yielded by the difference quotient using $h = 0.1$.

   (b) Write down an equation for the corresponding secant line.

   (c) Write down and equation for the precise tangent line at $x = 2$.

9. If f and g are differentiable function, prove that

$$\frac{d}{dx}(2f(x) + 3g(x)) = 2f'(x) + 3g'(x)$$

10. The complete graph of a function f is shown in figure 2; it consists of a line segment, a quarter-circle, and a semi-circle. Sketch the graph of f'.
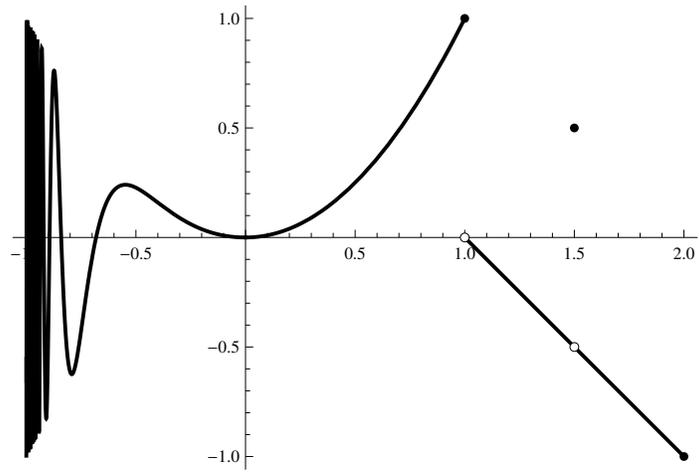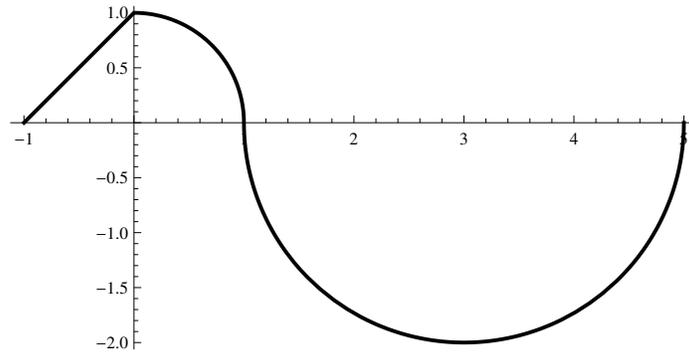
Figure 1: The graph for problem 5

Figure 2: The complete graph for problem 10