Calc I - Review for Exam III

There will be a 100 point exam this Friday, April 15 I would guess there will be 7 to 9 questions. Here are some problems to get you started thinking about it. Note that these are the *kinds* of problems you might expect to see. You have plenty of examples of similar types of problems, including your textbook and in class problem sheets. As always, we'll go over this on Wednesday but you need to have thought about these problems prior to that.

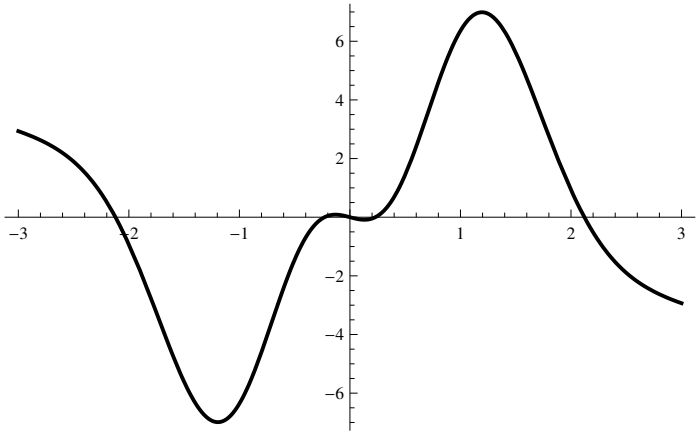

Figure 1: The graph of $f(x) = 20x^3e^{-x^2} - x$

1. The graph of $f(x) = 20x^3e^{-x^2} - x$ is shown in figure 1.
   (a) Write down an equation that the critical points of f must satisfy.
   (b) Suppose we wanted to find the smallest, positive critical point of f using Newton's method. Write down the corresponding Newton's method iteration function and a reasonable initial guess to start the iteration.
   (c) Find the exact values of the inflection points of f and indicate their positions in the graph.
2. Let $f(x) = 3x^4 - 4x^3 - 12x^2$
   (a) Find all the critical points of f.
   (b) Sketch a rough graph of f.
   (c) Find the absolute minimum value of f.
   (d) Find the locations of any other local extremes of f.
3. The top of a 15 foot long ladder slides down a wall at 3 feet per second. How fast is the bottom of the ladder moving away from the wall when it is 12 feet away from the wall?

4. Suppose I set up a rectangular corral with inner partitions, as shown in figure 2. The material for the exterior portion costs twice as much as the material for the interior walls. What are the dimensions of the cheapest such corral?

5. Let $f(x) = x^3 - x - 1$. Use two Newton steps from $x_0 = 1$ to find a good rational approximation to the root of f.

6. Find the anti-derivatives of the following functions.

   (a) $f(x) = x^3 - 2x^2 + 3x - 4$

   (b) $f(x) = e^x - \sin(x) + \cos(x)$

7. Suppose an object starts at position $p_0 = 1$ with initial velocity $v_0 = -1$ and moves with acceleration given by $a(t) = 2t + 1$.

   (a) Find a position function $p(t)$ for the object.

   (b) Where is the object at time $t = 10$ and how fast is it going?

8. The graph of a function f is shown in figure 3. Compute

$$\int_0^4 f(x)dx.$$

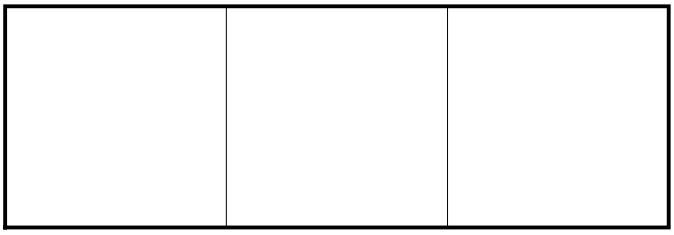

Figure 2: A corral

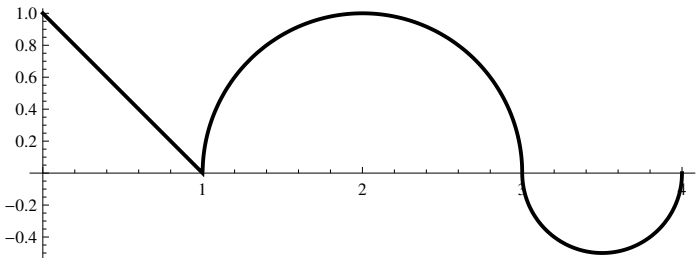

Figure 3: The plot of $f(x)$ for problem 8