Numerical Analysis - Review for exam I

The first exam will be this Friday, February 19. Here are some problems that might help.

1. Write down a careful statement of the following theorems.
   (a) The intermediate value theorem
   (b) The integral test with remainder
   (c) Taylor's theorem with remainder
2. A single precision (32-bit number) is with bit string $\boxed{s\,e\,m}$ is given by
$$(-1)^s \times 2^{e-127} \times (1.m)_2.$$
   Express the single precision machine numbers with the following bit strings in decimal form.
   (a) $\boxed{\texttt{0 10000001 01100000000000000000000}}$
   (b) $\boxed{\texttt{0 00000110 01010000000000000000000}}$
3. Express the number 42 in hexadecimal form.
4. Let $f(x) = e^x - x - 1$. In this problem, we'll consider two approaches to computing $f(0.01)$. Note that to five decimal digits, $e^{0.01} \approx 1.0101$.
   (a) Use the above approximation to compute $f(0.01)$.
   (b) Use the Taylor series of f to compute $f(0.01)$.
5. Let's consider the solution of the equation $\cos(x) = x^3$ using bisection. We'll rewrite it in terms of the function $f(x) = x^3 - \cos(x)$. Of course, a root of f is a solution of the original equation.
   (a) Use the intermediate value theorem to clearly explain why f has a root between $x = 0$ and $x = 2$.
   (b) Suppose we use the bisection method starting from the interval $[a, b] = [0, 2]$. Provide an upper bound on the number of iterations required to estimate the solution to within machine precision.
6. The function $f(x) = (x^2 + 2)/3$ has two fixed points: $x = 1$ and $x = 2$. Classify both points as attractive, repulsive, or neutral under iteration of f.
7. Suppose we wish to find a decent, rational approximation to $\sqrt[3]{3}$ using Newton's method. Noting that $\sqrt[3]{3}$ is a root of $f(x) = x^3 - 3$, write down the Newton's method iteration function N and perform two iterations from $x_1 = 1$.
8. Consider the data $(-1, 0)$, $(1, 1)$, $(2, 2)$.
   (a) Write down the unique quadratic that interpolates this data in Lagrange form.
   (b) Write down the unique quadratic that interpolates this data in factored Newton form.
   (c) Write down a system of linear equations that the natural cubic spline that fits this data must satisfy.