Numerical Analysis - Review 2

1. Compute the LU decomposition of the matrix

$$\begin{pmatrix} 1 & 3 \\ 5 & 10 \end{pmatrix}.$$

   Be sure to pivot, if necessary!

2. Suppose we'd like to find a least squares fit to the data

$$\{(1,2),(2,3),(4,4)\}$$

   using a function of the form $f(x) = ax + b$. Write down the normal equations that we'd need to solve to find this fit.

3. Suppose we'd like to estimate

$$\int_{-1}^{1} \cos(x^2)\, dx$$

   to within 10^{-6} of actual value using a trapezoidal sum.

   (a) How many terms do we need in our sum to ensure the error estimate?

   (b) Write down the resulting sum using summation notation.

   Note: The graph of f'' is shown on the reverse.

4. Estimate the solution to the IVP $y' = (t+1)y$; $y(0) = 1$ over the time interval $0 \le t \le 1$ using Euler's method with $\Delta t = 0.5$.

5. Consider the linear system $x' = x - y$, $y' = x + 2y$ with initial condition $x(0) = 0$, $y(0) = 1$. Apply Euler's method with a time step of $\delta t = 0.5$ to approximate the solution of this system over the time interval $0 \le t \le 1$.

6. Suppose we wish to approximate the solution to $y' = 10te^{-\left((y-1)^2+(y+1)^2+(t-1)^2\right)}$ over the interval $[0,4]$ to a global tolerance of 0.01. Write down an inequality that the step size h should satisfy and solve for h.
   Note: You may be interested in the graphs shown in figure 2.

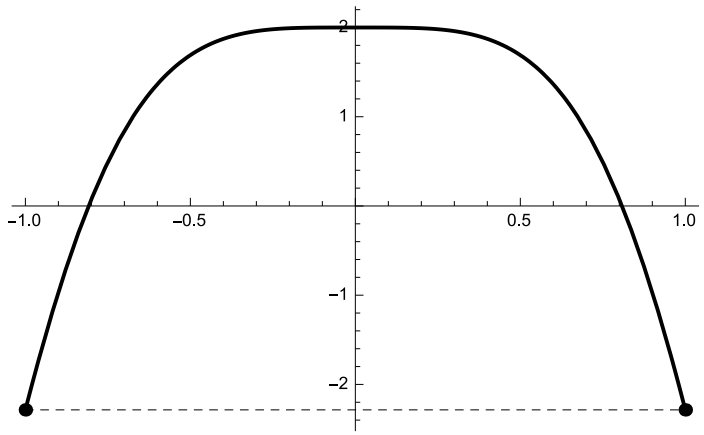

Figure 1: The plot of $f''(x)$ for problem 3

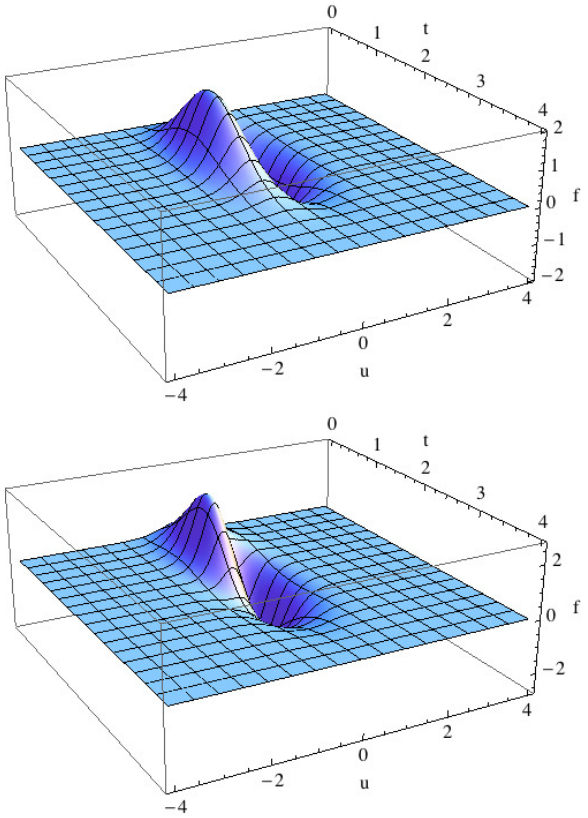

Figure 2: The graphs of f_y (top) and $f_t + ff_y$ (bottom) for the last problem