

Calc II - Review for exam I

The first exam will be next Friday, February 10. We will discuss some of these problems in class on Wednesday, but you should work them all out to the best of your ability prior to that. Understanding the problems on this sheet will help you greatly on the exam.

1. Evaluate the following integrals using the technique indicated.

(a) $\int \frac{x}{2x^2 + 1} dx$ - u -subs

(b) $\int x \cos(x) dx$ - by parts

(c) $\int_0^{2\pi} \cos^2(x) dx$ - with the $\frac{1}{2}$ -angle formula

(d) $\int_0^1 \frac{x^3}{\sqrt{1-x^2}} dx$ - trig subs

2. The complete graph of a function f is shown in figure 1 on the reverse - it consists of three straight line segments and a quarter circle. Evaluate $\int_{-2}^3 f(x) dx$.

3. Consider the integral $\int_0^2 x \sin(\cos(x^2 + 1)) dx$. Letting $u = x^2 + 1$, translate this integral to a new integral in terms of u . You need not evaluate the integral.

4. Suppose that

$$\int_a^b r f(r(x-s)) dx = \int_c^d f(u) du.$$

What are the values of c and d in terms of a and b ?

5. Evaluate the following integrals using any technique that you see fit.

(a) $\int_0^2 \sqrt{4-x^2} dx$

(b) $\int x \sqrt{4-x^2} dx$

(c) $\int x^2 \sqrt{4-x^2} dx$

(d) $\int_{-\pi}^{\pi} \frac{\sin^3(x)}{\cos^5(x)} dx$

(e) $\int e^{2x} \cos(3x) dx$

(f) $\int \frac{1}{x^2 \sqrt{4-x^2}} dx$

(g) $\int \frac{1}{(1+x^2)^2} dx$

(h) $\int \frac{1}{x(1+x^2)^2} dx$

(i) $\int \frac{1}{(x+1)(x-2)} dx$

6. Suppose we wish to estimate

$$\int_0^3 \sqrt{x^3 + 1} dx$$

with a midpoint sum and we'd like our result to be within 0.0001 of the actual value.

- Find an n large enough so that n terms will guarantee your estimate is within the desired accuracy.
- Write down the resulting sum using summation notation.

Note that the graph of $f(x) = \sqrt{x^3 + 1}$ together with its second derivative is show in figure 2 below.

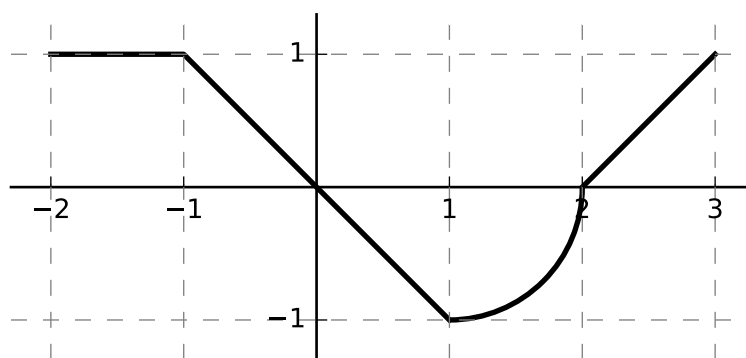


Figure 1: The complete graph of a function

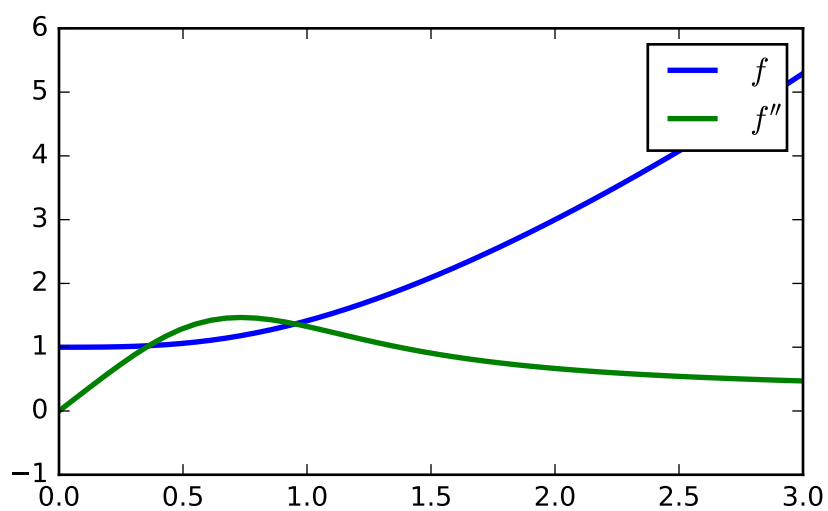


Figure 2: The graph of $f(x) = \sqrt{x^3 + 1}$ together with its second derivative