Calc II - Review for exam III

The third exam will be next Friday, April 21. We will discuss some of these problems in class on Wednesday, but you should work them all out to the best of your ability prior to that. Understanding the problems on this sheet will help you greatly on the exam.

1. Use the geometric series formula to express $0.21\overline{12}$ as a fraction.
2. Use the integral test to show that $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n(\ln(n))^p}$ converges precisely when $p > 1$.
3. Suppose we'd like to approximate
$$\sum_{n=1}^{\infty} \frac{n}{n^4+1}$$
by truncating the sum to obtain a finite sum of the form
$$\sum_{n=1}^{N} \frac{n}{n^4+1}.$$
How large does N have to be to ensure that our approximation is within 0.0001 of the actual value?
4. Write down a couple of complete sentences using the comparison test to show that
$$\sum_{n=1}^{\infty} \frac{\sin\left(n^3\right)}{n^4}$$
converges absolutely.
5. Write down a couple complete sentences using the alternating series test to show that
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln(n)}{n}$$
converges conditionally.
6. Use the ratio test to determine whether
$$\sum_{n=1}^{\infty} \frac{n^2}{n!}$$
converges.
7. Classify the following series as absolutely convergent, conditionally convergent, or divergent.
   (a) $\sum (-1)^n \dfrac{n^2}{n^2+2}$
   (b) $\sum (-1)^n \dfrac{n^2}{n^3+2}$
   (c) $\sum (-1)^n \dfrac{n^2}{n^4+2}$

8. Find the domain of convergence of the following power series.
   (a) $\sum(-1)^n \frac{n}{5^n} x^n$
   (b) $\sum(-1)^n \frac{1}{n5^n} x^n$
9. Find the radius of convergence of $\sum(-1)^n \frac{n!}{n^n} x^n$.
10. Starting with the geometric series formula, find a power series for $x/(1 + x^7)$, then find a series representation of
$$\int \frac{x}{1+x^7}\,dx.$$
11. Start a square of side length 1, divide it into 9 smaller squares of side length 3, and discard the square in the middle. That leaves you with 8 more smaller squares; do the same to each of those and continue the process indefinitely, as shown in figure 1. What is the total area of the squares removed in this process?

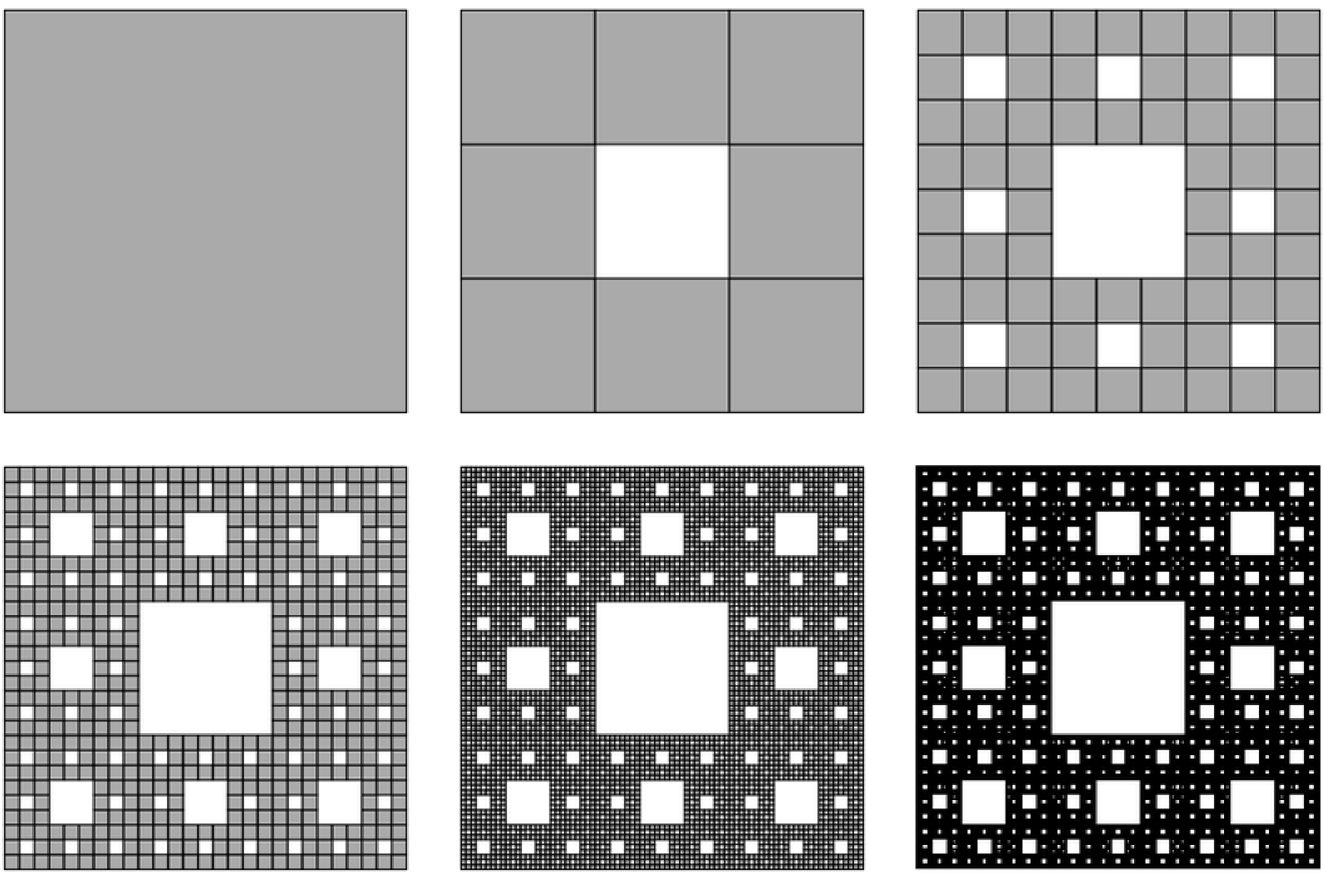

Figure 1: Constructing a fractal