# Advanced Calculus - Practice problems for exam 1

1. Let $D$ be the portion of $2x + 5y - z = 0$ inside $x^2 + y^2 = 1$ oriented up and let $\mathbf{F} = \langle y, z, -x \rangle$. Use Stokes's theorem to evaluate
$$\oint_C \mathbf{F} \cdot d\mathbf{T}.$$

2. Let $C$ denote the unit circle in the plane $z = 0$ oriented counter-clockwise about the $z$-axis. Compute
$$\oint_C x^2 z dx + 3x dy - y^3 dz$$
both directly from the definition of line integral and using Stoke's theorem.

3. Let $R$ be the rectangle in space with vertices $(0, 0, 0)$, $(2, 2, 0)$, $(2, 2, 1)$, $(0, 0, 1)$ and let $n$ denote the unit vector with positive $x$ component that is normal to this rectangle. Compute
$$\int_R \mathbf{F} \cdot d\mathbf{n}$$
for each of the following fields:
   (a) $\mathbf{F} = \langle x, y, z \rangle$
   Should be quite easy - no integration required, if you can visualize.
   (b) $\mathbf{F} = \mathbf{e}_\theta$ expressed in cylindrical coordinates
   Also relatively easy, no integration required, if you can visualize.
   (c) $\mathbf{F} = \langle z, y, x \rangle$
   I'm afraid you'll have to parametrize and integrate.

4. Let $\mathbf{F}(x, y, z) = \left(x^2 + y^2 + z^2\right)^2 \langle x, y, z \rangle$. In this problem, we'll consider
$$\int_S \mathbf{F} \cdot d\mathbf{n}$$
where $S$ is the surface of the unit sphere.

(a) Use the basic interpretation of flux to explain why the integral must evaluate to $4\pi$.

(b) Use the divergence theorem to express the integral as a triple integral in terms of the spherical coordinates $\rho$, $\varphi$, and $\theta$.

(c) Evaluate your integral from part (b).

5. Let $\mathbf{F}(\rho, \varphi, \theta) = \rho\varphi\mathbf{e}_\rho + \varphi\mathbf{e}_\theta$. Compute

$$\iint_S \mathbf{F} \cdot d\mathbf{n},$$

where $S$ is the surface of the unit sphere.

6. Let $\mathbf{F}(r, \theta) = r\mathbf{e}_r + e^{-r^2}\mathbf{e}_\theta$. Compute

$$\oint_C \mathbf{F} \cdot d\mathbf{n},$$

where $C$ is the unit circle.