Calc I - Comments on the Final Exam

Our final exam is this coming Friday, May 4 at 8:00 in our usual class room. It will be cumulative but the recent material on integration feature prominently. In addition to the problems on integration, you should revisit some select problems from past exams. For your convenience, I've included those problems here.

Integration

1. Use the Fundamental Theorem of Calculus to evaluate the following definite integrals
   (a) $\displaystyle\int_0^2 (3x^2 + x + 2)\, dx$
   (b) $\displaystyle\int_1^e \left(\frac{1}{x^2} + \frac{1}{x}\right) dx$
2. In this problem, we'll explore the relationship between a definite integral of the sine function and the area between the graph of the sine and the x-axis.
   (a) Sketch the graph of $f(x) = \sin(x)$.
   (b) Evaluate $\displaystyle\int_0^{\pi} \sin(x)\, dx$.
   (c) Evaluate $\displaystyle\int_0^{2\pi} \sin(x)\, dx$.
   (d) What is the area between the graph of the sine function and the interval $[0, 2\pi]$ on the x-axis?
3. The complete graph of a function is shown in figure 4; it consists of two line segments and a quarter circle. Evaluate
$$\int_{-2}^{4} f(x)dx.$$
4. Write down a right Riemann sum with $n = 5$ terms to estimate
$$\int_0^1 \sin(x)\, dx.$$

Exam 1

3. Let
$$f(x) = \frac{(3x-3)(x+2)}{(4x+8)(x-3)}.$$
Evaluate each of the following the limits. You do not need to show work.
   (a) $\lim_{x\to 3^-} f(x)$
   (b) $\lim_{x\to 3^+} f(x)$
   (c) $\lim_{x\to 3} f(x)$
   (d) $\lim_{x\to -2} f(x)$
   (e) $\lim_{x\to \infty} f(x)$
4. Evaluate each of the following the limits, showing your work carefully.
   (a) $\lim\limits_{x\to 3} \dfrac{x-3}{x^3-4x^2+2x+3}$
   (b) $\lim\limits_{x\to 0} \dfrac{x}{\sqrt{x+4}-2}$
7. Find the derivative of $f(x) = x^3 - 2$, *using the definition of the derivative.*
8. Figure 1 shows the complete graph of a function together with a spare set of axes. Sketch a graph of f' on that spare set.

Exam 2

1. Use the differentiation rules to find the derivative of each of the following functions.
   (a) $f(x) = x^{42} - 2x^3 + x^2 - 4x$
   (b) $f(x) = e^x + \sin(x) + \cos(x)$
   (c) $f(x) = x\sin(x)$
   (d) $f(x) = e^{-x^2}$
   (e) $f(x) = x^2\sqrt{x^4+1}$
   (f) $f(x) = x^2 + \sin^5(x)$
   (g) $f(x) = \dfrac{1}{x^2}\sin(x)$
   (h) $f(x) = \dfrac{x^6 + \frac{1}{x}}{\sin(x)}$
2. Let $f(x) = 2^x$.
   (a) Write down the difference quotient for f.
   (b) Use a little algebra to separate the xs in your expression from the hs. You should get an expression in x times a limit involving h.
   (c) Using table 1 at the top of the next page, write down a good estimate to $f'(x)$.

Table 1: Values of the difference quotient for 2^x

h	0.1	0.01	0.001	0.0001	0.00001
$\frac{2^h-1}{h}$	0.717735	0.695555	0.693387	0.693171	0.69315

5. Sketch the graph of $f(x) = -2\sin\left(\frac{1}{3}x + \pi\right)$. Be sure to label the x-intercepts as well as the x and y coordinates of the maxima and minima.
6. Let $f(x) = x^2\cos(2x)$. Find an equation for the line tangent to the graph of f at the point $(\pi/2, f(\pi/2))$.

Exam 3

1. Compute the derivatives of the following functions.
   (a) $f(x) = \ln(x^2) + e^{2x}$.
   (b) $f(x) = \arctan(2x+1)$.
   (c) $f(x) = x^2\arcsin(x^3)$.
3. Find an equation of the line that is tangent to the curve satisfying $x^2 + x^2y - y^3 = 1$ at the point $(1, 1)$.
4. Let $f(x) = 2x^3 - 3x^2 - 12x + 4$
   (a) Find all the critical points of f.
   (b) Sketch a rough graph of f using the information from part (a).
   (c) Label any relative maxima or minima on the graph including both x and y coordinates.
   (d) Find the exact locations of any inflection points of f and label them on the graph a well.
6. The bottom of a 12 foot long ladder slides away from a wall at 2 feet per second. How fast is the top of the ladder moving towards the floor when it is 3 feet above from the floor?
7. I have a piece of cardboard that measures 3 feet by 2 feet. I'd like to cut the corners out of it as shown in figure 2 so that I can fold up the sides and make a box with no top.
   (a) How large a square should I cut out of each corner so that the resulting box has the largest possible volume?
   (b) What is the largest possible volume that the box can have?

One more applied problem

1. We wish to construct a corral that is partitioned into three parts as shown in figure 3. The thicker exterior fence costs \$20/foot while the thinner interior fence cost \$10/foot. The total amount enclosed is to be $3000\,\text{ft}^2$.
   (a) What is the cost of the least expensive corral that will work?
   (b) How much of each type of fence (interior and exterior) should we purchase?

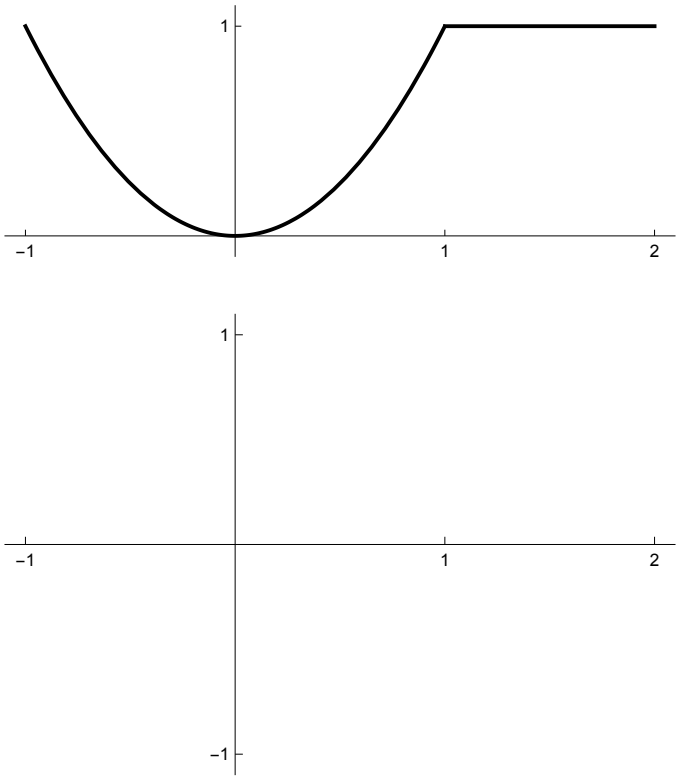

Figure 1: The complete graph of a function together with a spare set of axes

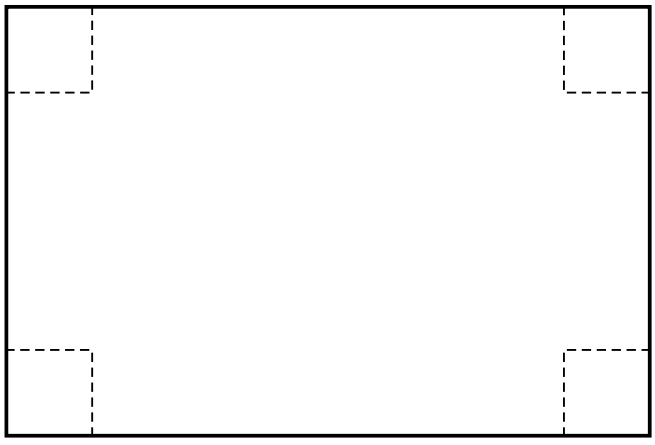

Figure 2: A a 3×2 piece of cardboard with corners to cut out.

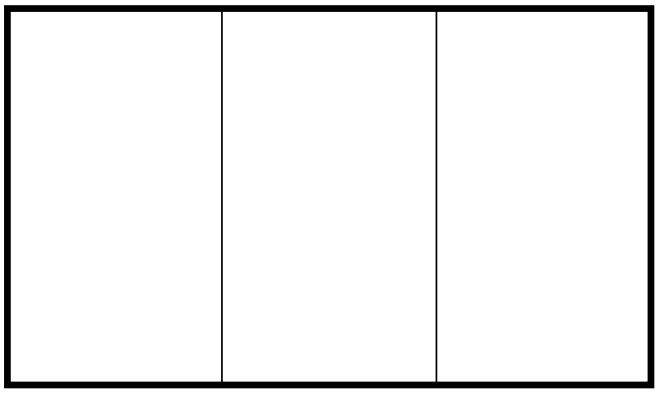

Figure 3: A partitioned corral

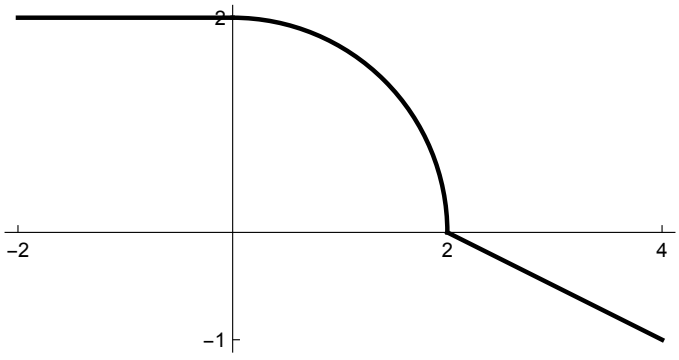

Figure 4: The complete graph of a function