Calc I - Review for exam 2

The second exam will be this Friday, March 9. We will discuss some of these problems in class on Wednesday, but you should work them all out to the best of your ability prior to that. Understanding the problems on this sheet will help you greatly on the exam.

1. Use the differentiation rules to find the derivative of each of the following functions.

   (a) $f(x) = x^2 + \dfrac{1}{x}$

   (b) $f(x) = e^x + \dfrac{1}{x}$

   (c) $f(x) = \dfrac{x^5}{\sin(x)}$

   (d) $f(x) = x^7\left(x^2 + x\right)$

   (e) $f(x) = \sqrt{\sin(x)} + \cos(x)$

   (f) $f(x) = \dfrac{1}{x^7}\cos\left(x^7\right)$

   (g) $f(x) = x^4\cos^2(x)$

   (h) $f(x) = \dfrac{e^x}{x^3}\left(x + \dfrac{1}{x}\right)$

   (i) $f(x) = \dfrac{4^{-x}}{x^4}$

   (j) $f(x) = \dfrac{e^{-x}\cos(x)}{\sin(\cos(x))}$

2. Let $f(x) = 8^x$.

   (a) Write down the difference quotient for f.

   (b) Use a little algebra to separate the xs in your expression from the hs. You should get an expression in x times a limit involving h.

   (c) Using the table below, write down a good estimate to $f'(x)$.

h	0.1	0.01	0.001	0.0001	0.00001
$\dfrac{8^h - 1}{h}$	2.31144	2.10121	2.08161	2.07966	2.07946

3. Use the definition of the derivative to prove each of the following differentiation rules.

   (a) $\dfrac{d}{dx}(af(x) + bg(x)) = af'(x) + bg'(x)$

   (b) $\dfrac{d}{dx}x^2 f(x) = 2xf(x) + x^2 f'(x)$

4. Compute the following limits using the fact that $\lim_{\theta \to 0} \dfrac{\sin(\theta)}{\theta} = 1$.

   (a) $\lim_{x \to 0} \dfrac{\sin(3x)}{x}$

   (b) $\lim_{x \to 0} \dfrac{\sin(3x)}{\sin(2x)}$

   (c) $\lim_{x \to 0} \dfrac{1 - \cos^2(x)}{2x^2}$

5. Sketch the graphs of each of the following functions.

   (a) $f(x) = \sin(2x - \pi)$

   (b) $f(x) = 2\cos(\pi x) + 2$

6. In this problem, we'll use the pre-drawn axes in figure 1 to draw the graphs of $f(x) = \sin(2x)$ and $f'(x) = 2\cos(2x)$ and explore their relationship.

   (a) Sketch the graph of $f(x) = \sin(2x)$ on the top pair of axes.

   (b) Sketch the graph of $f'(x) = 2\cos(2x)$ on the bottom pair of axes.

   (c) Identify all points with horizontal tangent lines on the top graph. Projecting down, does the bottom graph cross the x axis at those points?

7. Let $f(x) = x\sin(x)$. Find an equation for the line tangent to the graph of f at the point $(\pi/2, f(\pi/2)$.

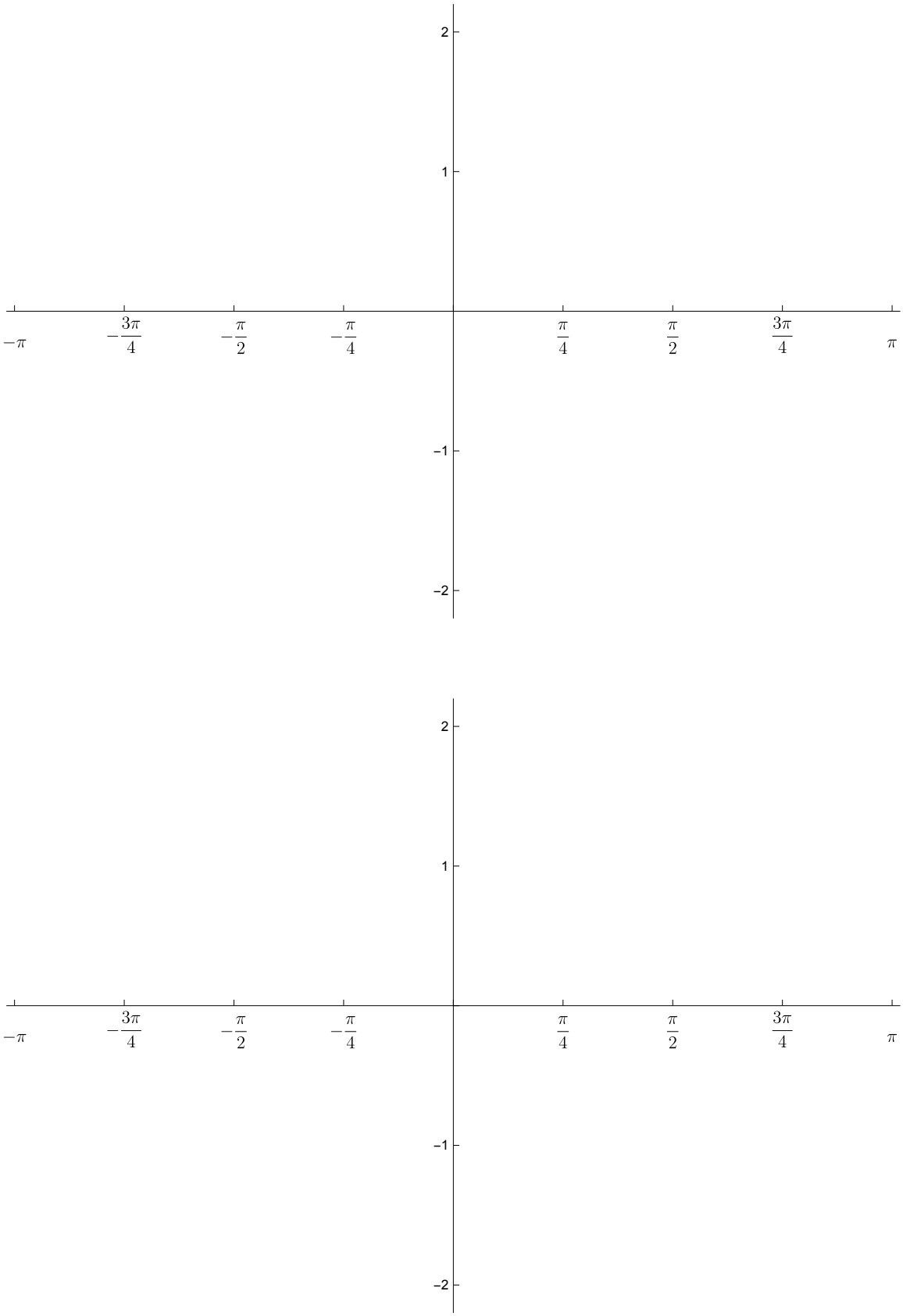

Figure 1: A pair of axes for trig functions