# Calc I - Review for Exam III

There will be an exam This Friday, April 13 and many of the problems will be like something on this review sheet.

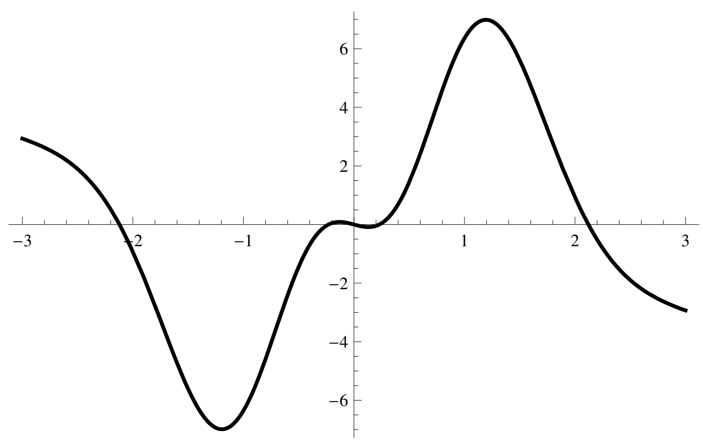


Figure 1: The graph of $f(x) = 20x^3e^{-x^2} - x$

1. A few more derivatives
2. Find an equation of the line that is tangent to the curve satisfying $x^3 - 3x^2y^3 + y^4 = -1$ at the point $(1, 1)$.
3. In this problem, we're going to derive the fact that, if $f(x) = \ln(2x)$, then $f'(x) = 1/x$ using the fact that we know the inverse of $f$.
   (a) Starting from $y = \ln(2x)$, write the equation in exponential form.
   (b) Implicitly differentiate your equation from part (a) with respect to $x$.
   (c) Solve your equation from part (b) for $y'$.
   (d) Simplify, if necessary to show that $y' = 1/x$.

4. The graph of $f(x) = 20x^3 e^{-x^2} - x$ is shown in figure [fig:exp].
    (a) Write down an equation that the critical points of $f$ must satisfy.
    (b) Suppose we wanted to find the smallest, positive critical point of $f$ using Newton's method. Write down the corresponding Newton's method iteration function and a reasonable initial guess to start the iteration.
    (c) Find the exact values of the inflection points of $f$ and indicate their positions in the graph.
5. Let $f(x) = 3x^4 - 4x^3 - 12x^2$
    (a) Find all the critical points of $f$.
    (b) Sketch a rough graph of $f$.
    (c) Find the absolute minimum value of $f$.
    (d) Find the locations of any other local extremes of $f$.
6. The top of a 15 foot long ladder slides down a wall at 3 feet per second. How fast is the bottom of the ladder moving away from the wall when it is 12 feet away from the wall?
7. Suppose I set up a rectangular corral to enclose 1000 square feet with inner partitions, as shown in figure [fig:fence]. The material for the exterior portion costs twice as much as the material for the interior walls. What are the dimensions of the cheapest such corral?
    Comment: There's a good chance that a familiar optimization problem - like this one or a pizza box type problem will be on the exam.
8. Let $f(x) = x^3 - x - 1$. Use two Newton steps from $x_0 = 1$ to find a good rational approximation to the root of $f$.

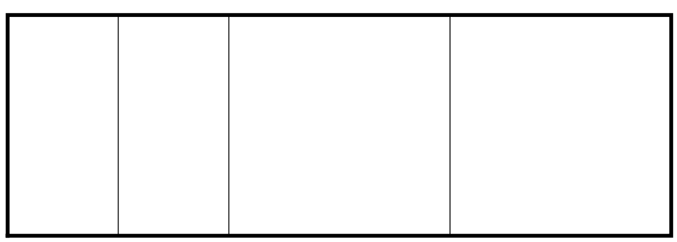

Figure 2: A corral