Calc I - Comments on the Final Exam

Our final exam is this coming Friday, May 3 at 8:00 AM for morning folks and on Monday, May 6 at 11:30 AM for afternoon folks. Both exams will be in our usual classroom. in our usual class room. It will be cumulative but the recent material on integration feature prominently. In addition to the problems on integration, you should revisit some select problems from past exams. For your convenience, I've included those problems here.

Note that for reasons discussed with the afternoon section, the final exam will count *double*. Thus, I'd expect it to in the neighborhood of 300 total points. Note also that I will hold all students to the highest standards of academic honesty. Anyone caught cheating on this final exam will receive a failing grade for the class.

Exam 1

4. Find the derivatives of the following functions, *using the definition of the derivative.*
   (a) $f(x) = 3x^2 - 2x + 1$
   (b) $f(x) = 2x^6$
5. Find the derivatives of the following functions, using the power rule, sum rule, and/or constant multiple rule.
   (a) $f(x) = 3x^2 - 2x + 1$
   (b) $f(x) = \sqrt{x}$
   (c) $f(x) = \sqrt{x}(3x^2 - 2x + 1)$
   (d) $f(x) = (x - 5)^2$
6. Let $f(x) = x^3 - 8x$.
   (a) Find the derivative of f.
   (b) Write down an equation for the tangent line at $x = 3$.
8. The complete graph of a function f is shown in figure 1. Sketch the graph of f'.

Exam 2

1. Use the differentiation rules to find the derivative of each of the following functions.
   (a) $f(x) = x^2 + \sin(x) - \cos(x) + e^x + \ln(x) + \tan(x)$
   (b) $f(x) = x^3 \sin(x)$

(c) $f(x) = \cos(x^3)$

(d) $f(x) = \dfrac{\sin(x)}{x^2+1}$

(e) $f(x) = \dfrac{x\ln(x)}{\cos(e^x)}$

2. Let $f(x) = 3^x$.

   (a) Write down the difference quotient for f.

   (b) Use a little algebra to separate the xs in your expression from the hs. You should get an expression in x times a limit involving h.

   (c) Using the table below, write down a good estimate to $f'(x)$.

h	0.1	0.01	0.001	0.0001	0.00001
$\dfrac{3^h-1}{h}$	1.1612317	1.104669	1.099216	1.098672	1.098618

3. Use the definition of the derivative to prove that

$$\frac{d}{dx}xf(x) = f(x) + xf'(x).$$

6. Consider the equation $x^3 - y^3 = xy^2 + 1$.

   (a) Identify which of the following points lies on the graph of the equation.

   - $(1, 1)$
   - $(1, -1)$

   (b) For the point or points above that do lie on the graph, find an equation of the line that is tangent to the graph.

Exam 3

1. Use the differentiation rules to compute the derivatives of the following functions.

   (a) $f(x) = \arctan(x) + \arcsin(x)$

   (b) $f(x) = x^4 \arctan(x)$

   (c) $f(x) = \arcsin(3x^2)$

   (d) $f(x) = x^3 \arcsin(x) \arctan(x^2)$

2. Figure 2 shows the graph of

$$f(x) = 2x + \sin(x) + 5.$$

   (a) Sketch the graph of f^{-1}.

   (b) Evalaute $(f^{-1})'(5)$.

5. Figure 3 shows the graph of

$$f(x) = x^2 e^{-x}.$$

   (a) Find the exact locations of the absolute maximum and minimum values.

   (b) Find the exact locations of the inflection points.

7. I'd like to make a box without a lid by cutting the corner's out of a 12×12 inch piece of cardboard and folding the sides up.
   (a) What is the maximum volume of such a box?
   (b) How big a corner should I cut out to create the box?

One more applied problem

1. We wish to construct a corral that is partitioned into four parts as shown in figure 4. The thicker exterior fence costs \$20/foot while the thinner interior fence cost \$10/foot. The total amount enclosed is to be $3000\,\text{ft}^2$.
   (a) What is the cost of the least expensive corral that will work?
   (b) How much of each type of fence (interior and exterior) should we purchase?

New stuff!!

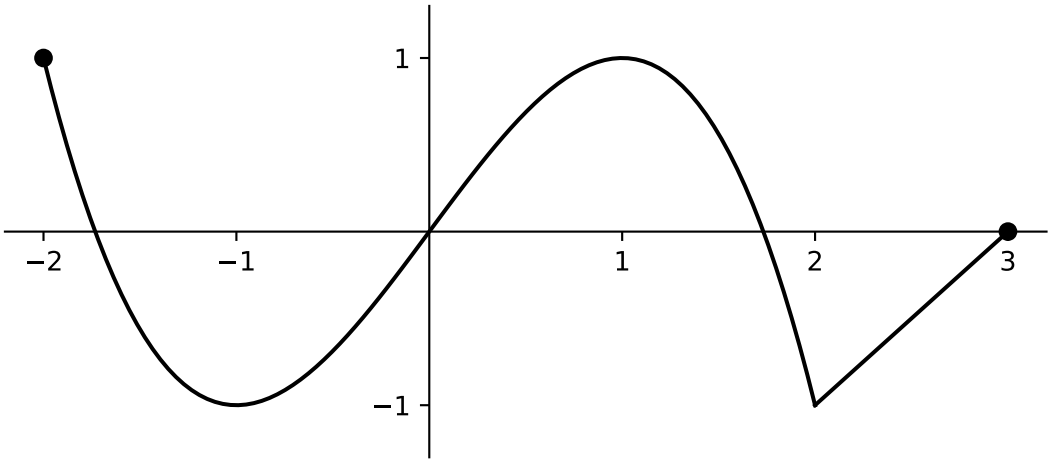
1. Let $f(x) = x^4 - 5$.
   (a) Write down the Newton's method iteration function for f.
   (b) Take two Newton steps for f from $x_1 = 1$.
2. Suppose the volume of a sphere changes at the rate of 4 cubic centimeters per second. At what rate is the radius of the sphere increasing when it's 5 centimeters?
3. Suppose I pull the bottom of a 10 foot tall ladder away from a wall at the rate of 2 feet per second. At what rate is the top of the ladder moving towards the floor when it is 3 feet away from the floor?
4. The velocity of an object is given by $v(t) = 2t - 1$ and, at time zero, the posistion of the object is $p_0 = 1$. Find the position $p(t)$ of the object as a function of time.
5. Use the Fundamental Theorem of Calculus to evaluate the following definite integrals
   (a) $\displaystyle\int_0^2 (3x^2 + x + 2)\,dx$
   (b) $\displaystyle\int_1^e \left(\frac{1}{x^2} + \frac{1}{x}\right) dx$
6. In this problem, we'll explore the relationship between a definite integral of the sine function and the area between the graph of the sine and the x-axis.
   (a) Sketch the graph of $f(x) = \sin(x)$.
   (b) Evaluate $\displaystyle\int_0^{\pi} \sin(x)\,dx$.
   (c) Evaluate $\displaystyle\int_0^{2\pi} \sin(x)\,dx$.
   (d) What is the area between the graph of the sine function and the interval $[0, 2\pi]$ on the x-axis?

7. The complete graph of a function is shown in figure 5; it consists of two line segments and a quarter circle. Evaluate

$$\int_{-2}^{4} f(x)dx.$$

8. Write down a right Riemann sum with $n = 5$ terms to estimate

$$\int_{0}^{1} \sin(x)\, dx.$$



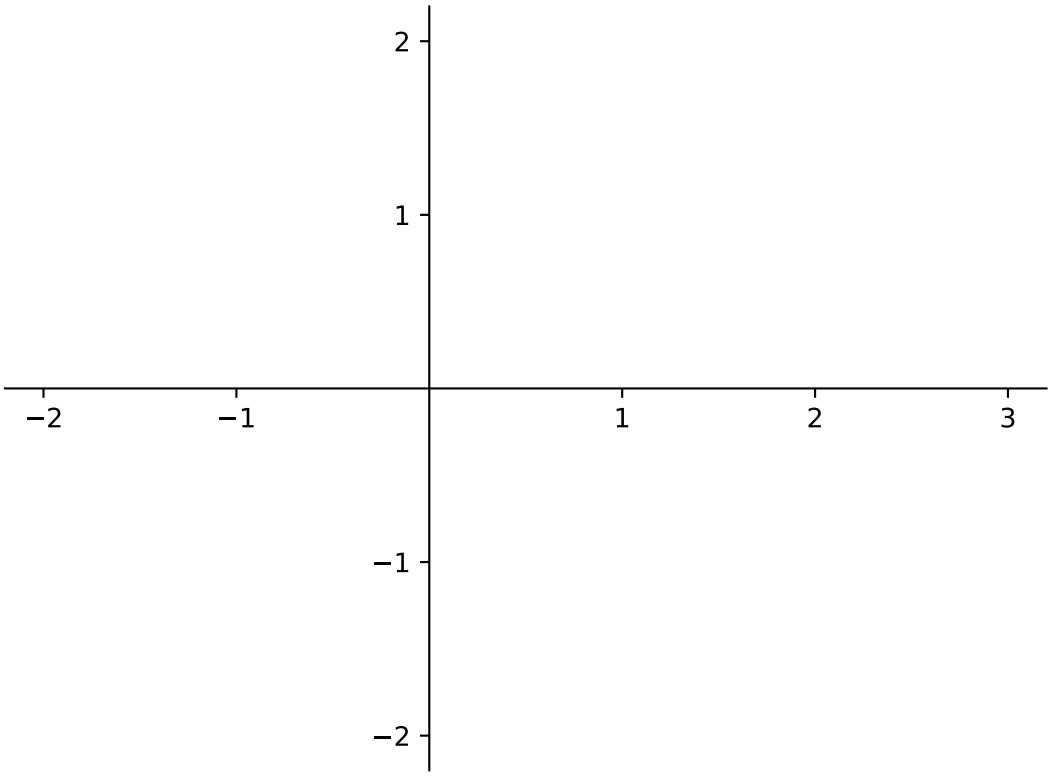


Figure 1: The graph of a function and a spare pair of axes

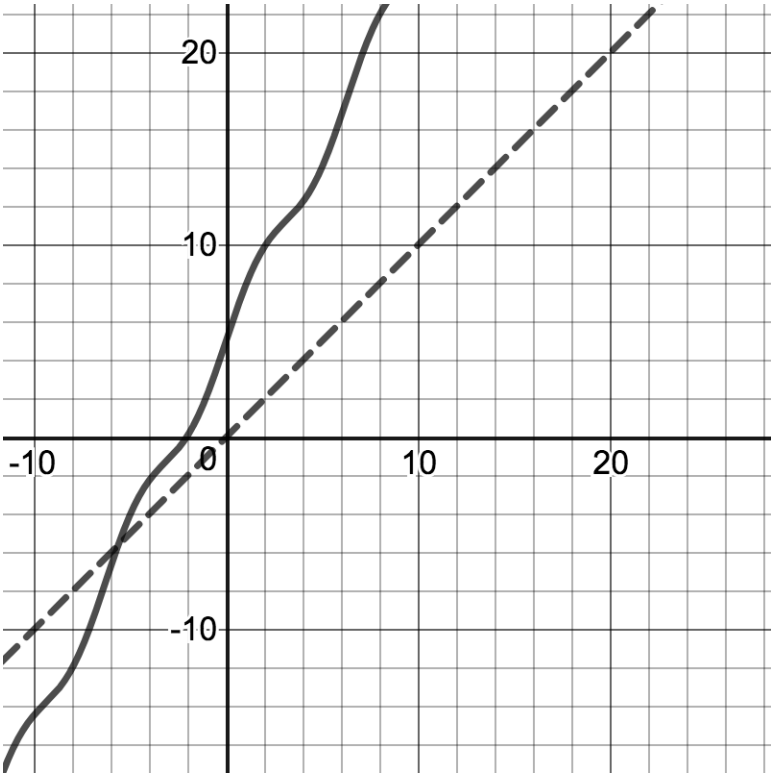


Figure 2: The graph of $2x + \sin(x) + 5$

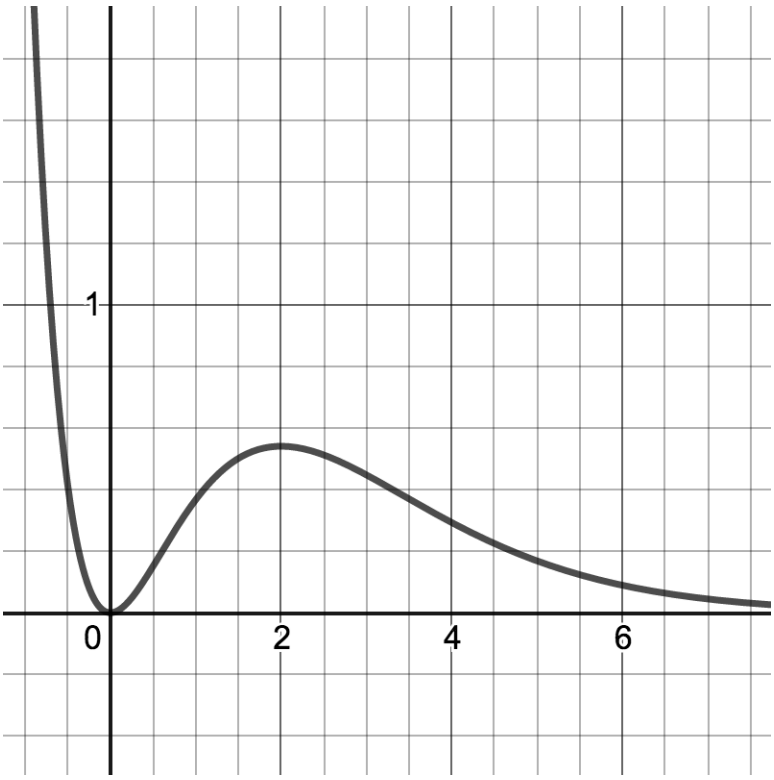


Figure 3: The graph of $f(x) = x^2 e^{-x}$

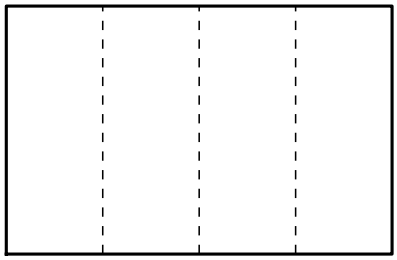


Figure 4: A partitioned corral

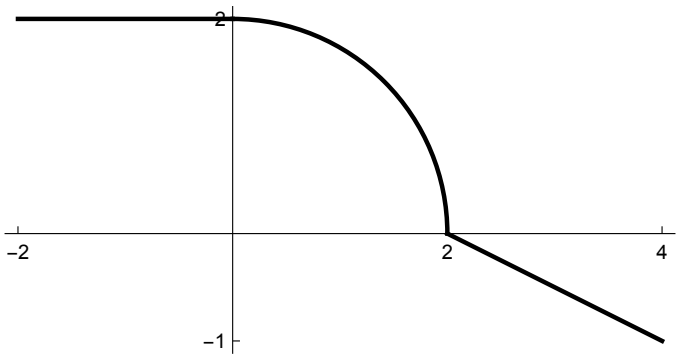


Figure 5: The complete graph of a function