Calc I - Review for exam I

The first exam will be next Friday, February 8. Here are some problems that might help.

1. We'd like to estimate
$$\lim_{t \to 0} \frac{7^t - 1}{t}$$

   so we generate the following table:

t	0.1	0.01	0.001	0.0001	0.00001
$f(t)$	2.14814	1.96497	1.94780	1.94610	1.94593

   Write down the value of the limit to as many decimal places that you are confident of.

2. Let $f(x) = x^2 + 2x - 1$.

   (a) Sketch the graph of f, together with the line tangent to the graph at the point $(0, f(0))$.

   (b) Compute the slope of the secant line through the points $(0, f(0))$ and $(2, f(2))$. Draw this line on your graph from part (a).

   (c) Write down the difference quotient for f at 0, i.e. the slope of the secant line through the points $(0, f(0))$ and $(0 + h, f(0 + h))$.

   (d) Compute the limit as $h \to 0$ of your answer to part (c).

   (e) How does your answer to part (d) relate to your sketch from part (a)?

3. Let
$$f(x) = \frac{(5x - 1)(x - 3)}{x^2 - 9}.$$

   Evaluate each of the following the limits.

   (a) $\lim_{x \to -1} f(x)$

   (b) $\lim_{x \to -3^-} f(x)$, $\lim_{x \to -3^+} f(x)$, and $\lim_{x \to -3} f(x)$

   (c) $\lim_{x \to 3} f(x)$

4. Figure 1 shows the complete graph of a function f; its domain is $(-1, 2]$.

   (a) What are $f(1.5)$ and $\lim_{x \to 1.5} f(x)$?

   (b) What are $\lim_{x \to 1^-} f(x)$, $\lim_{x \to 1^+} f(x)$, and $\lim_{x \to 1} f(x)$?

   (c) What can you say about $\lim_{x \to -1^+} f(x)$.

5. Find the derivatives of the following functions, *using the definition of the derivative.*

   (a) $f(x) = 2x^2 - 4x$

   (b) $f(x) = 1/\sqrt{x}$

   (c) $f(x) = x^5$

6. Find the derivatives of the following functions, using the power rule, sum rule, and/or constant multiple rule.

   (a) $f(x) = 2x^2 - 4x$

   (b) $f(x) = 1/\sqrt{x}$

   (c) $f(x) = x^5$

   (d) $f(x) = x^5 \left(x^2 - x - 1\right)$

   (e) $f(x) = \left(x^2 - x - 1\right)/x$

   (f) $f(x) = (x - 5)^2$

   (g) $f(x) = (x + 2)^3$

7. Let $f(x) = x^2 - x$.

   (a) Find the derivative of f.

   (b) Write down and equation for the tangent line at $x = 2$.

8. If f and g are differentiable function, prove that

$$\frac{d}{dx}(2f(x) + 3g(x)) = 2f'(x) + 3g'(x)$$

9. Write down a complete sentence referring to the intermediate value theorem showing that the function $f(x) = x^7 + x^3 + x + 1$ has a root in the interval $(-1, 1)$.

10. The complete graph of a function f is shown in figure 2; it consists of a line segment, a quarter-circle, and a semi-circle. Sketch the graph of f'.
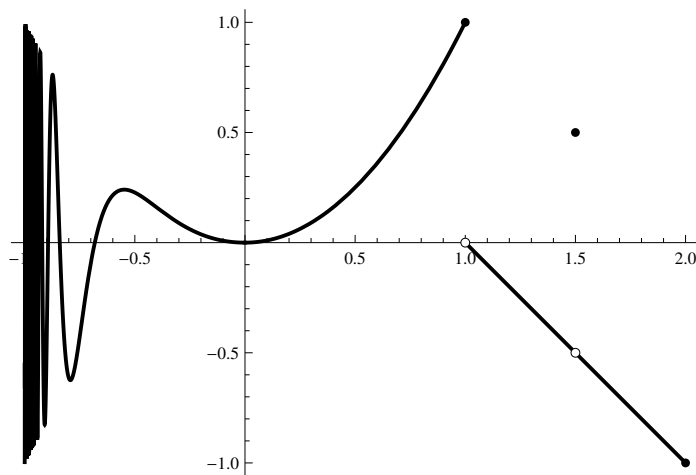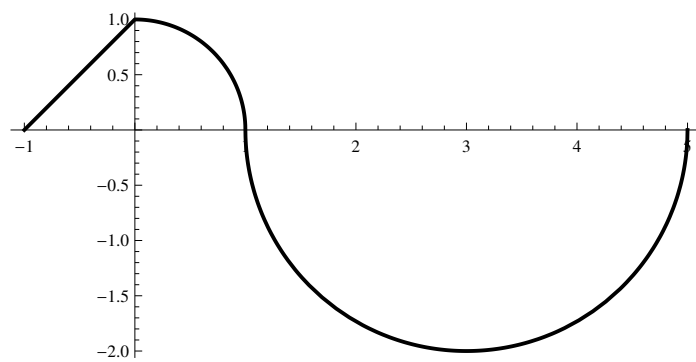

Figure 1: The graph for problem 6

Figure 2: The graph for problem 11