Calc I - Review for exam 2

The second exam will be this Friday, March 8. We will discuss some of these problems in class on Wednesday, but you should work them all out to the best of your ability prior to that. Understanding the problems on this sheet will help you greatly on the exam.

1. Use the differentiation rules to find the derivative of each of the following functions.

   (a) $f(x) = x^2 + \dfrac{1}{x}$

   (b) $f(x) = e^x + \dfrac{1}{x} + \ln(x)$

   (c) $f(x) = x^3 \ln(x)$

   (d) $f(x) = x \ln(x^3)$

   (e) $f(x) = \dfrac{x^5}{\sin(x)}$

   (f) $f(x) = x^7 \left(x^2 + x\right)$

   (g) $f(x) = \sqrt{\sin(x)} + \cos(x)$

   (h) $f(x) = \ln\left(\dfrac{1}{x^7}\right) \cos\left(x^7\right)$

   (i) $f(x) = x^4 \cos^2(x)$

   (j) $f(x) = \dfrac{e^x}{x^3}\left(x + \dfrac{1}{x}\right)$

   (k) $f(x) = \dfrac{4^{-x}}{x^4}$

   (l) $f(x) = \dfrac{e^{-x}\cos(x)}{\sin(\cos(x))}$

2. Let $f(x) = 8^x$.

   (a) Write down the difference quotient for f.

   (b) Use a little algebra to separate the xs in your expression from the hs. You should get an expression in x times a limit involving h.

   (c) Using the table below, write down a good estimate to $f'(x)$.

h	0.1	0.01	0.001	0.0001	0.00001
$\dfrac{8^h - 1}{h}$	2.31144	2.10121	2.08161	2.07966	2.07946

3. Use the definition of the derivative to prove each of the following differentiation rules.

   (a) $\dfrac{d}{dx}(af(x) + bg(x)) = af'(x) + bg'(x)$

   (b) $\dfrac{d}{dx}x^2 f(x) = 2xf(x) + x^2 f'(x)$

4. Compute the following limits using the fact that $\lim\limits_{\theta \to 0} \dfrac{\sin(\theta)}{\theta} = 1$.

   (a) $\lim\limits_{x \to 0} \dfrac{\sin(3x)}{x}$

   (b) $\lim\limits_{x \to 0} \dfrac{\sin(3x)}{\sin(2x)}$

   (c) $\lim\limits_{x \to 0} \dfrac{1 - \cos^2(x)}{2x^2}$

5. Sketch the graphs of each of the following functions.

   (a) $f(x) = \sin(2x - \pi)$

   (b) $f(x) = 2\cos(\pi x) + 2$

6. Here's a fun fact about polynomials: *A polynomial of degree n can have at most n roots.* For example, the top graph in figure 1 show a degree 5 polynomial p and it crosses the x-axis 5 times.

   (a) Sketch the graph of p' on the spare pair of axes.

   (b) How many times does your graph cross the x-axis?

   (c) How does this relate to the power rule?

7. Let $f(x) = x\sin(x)$. Find an equation for the line tangent to the graph of f at the point $(\pi/2, f(\pi/2))$.

8. Consider the equation $x^3 + y^3 = x^2 + 12$.

   (a) Identify which of the following points lies on the graph of the equation.

   - $(1, 1)$
   - $(1, 2)$
   - $(2, 2)$
   - $(2, 1)$

   (b) For the point or points above that do lie on the graph, find an equation of the line that is tangent to the graph.
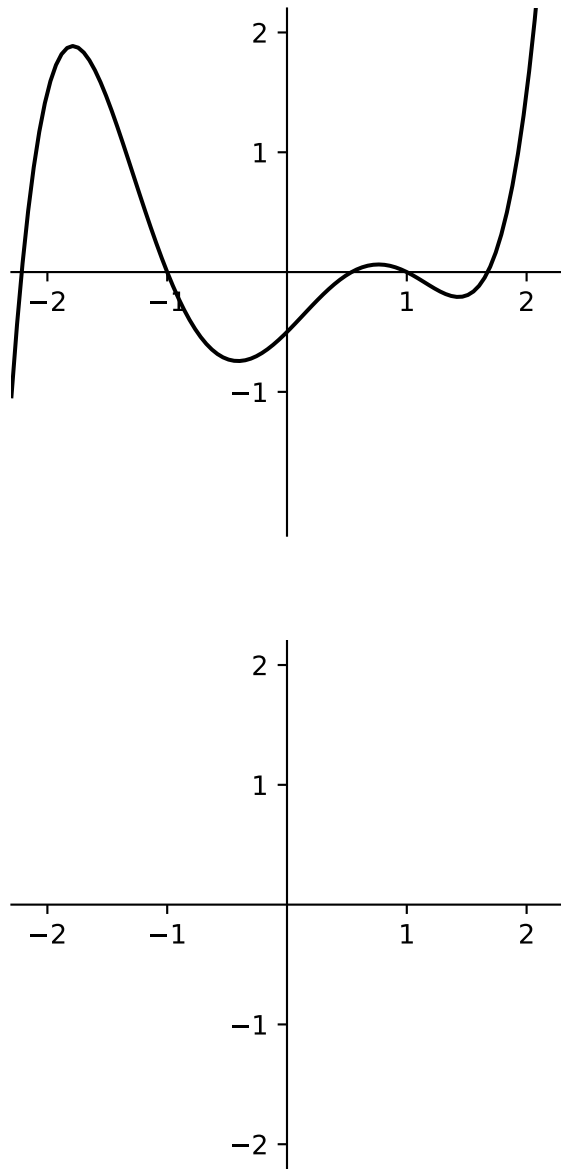
Figure 1: The graph of a quintic polynomial with a spare pair of axes