Calc I - Review for exam 3

The third exam and final exam during the term will be this Friday, April 12. We will discuss some of these problems in class on Wednesday, but you should work them all out to the best of your ability prior to that. Understanding the problems on this sheet will help you greatly on the exam.

1. Use the differentiation rules to compute the derivatives of the following functions.
   (a) $f(x) = \arctan(x) + \arcsin(x)$
   (b) $f(x) = x^3 \arcsin(x)$
   (c) $f(x) = \arctan(x^3)$
   (d) $f(x) = x \arcsin(x^2) \arctan(x^3)$
2. Figure 1 shows the graph of
$$f(x) = e^x + \frac{1}{x^2+1}.$$
   (a) Sketch the graph of f^{-1}.
   (b) Evalaute $(f^{-1})'(2)$.
3. Let $f(x) = e^x - 2x$. Find and classify the extremes of f on the interval $[0, 3]$.
4. Let $f(x) = x^3 - 2x^2$. Find the point or points guaranteed the mean value theorem over the interval $[0, 3]$.
5. Figure 2 shows the graph of
$$f(x) = x^3 e^{-x^2}.$$
   (a) Find the exact locations of the absolute maximum and minimum values.
   (b) Find the exact locations of the inflection points.
6. Let $f(x) = x^4 + x^3 - 7x - 1$.
   (a) Find the maximal intervals on which f is increasing and decreasing.
   (b) Find the the maximal intervals on which f is concave up or concave down.
   (c) Sketch a graph of f that clearly incorporates the information from parts (a) and (b).
7. Let $f(x) = x\sqrt{x-1}$.
   (a) What is the domain of f?
   (b) Find the maximal intervals on which f is increasing and decreasing.
   (c) Find the the maximal intervals on which f is concave up or concave down.
   (d) Sketch a graph of f that clearly incorporates the information from parts (a), (b), and (c).
8. I'd like to make a box without a lid by cutting the corner's out of a 12×20 inch piece of cardboard and folding the sides up.
   (a) What is the maximum volume of such a box?
   (b) How big a corner should I cut out to create the box?
9. I'd like to construct a corall to enclose a total area of 1240 square feet. The corall should be divided into three subsections as shown in figure 3. The fence for the exterior portion costs three times as much as the fence for the interior. What are the dimensions of the cheapest such corall?

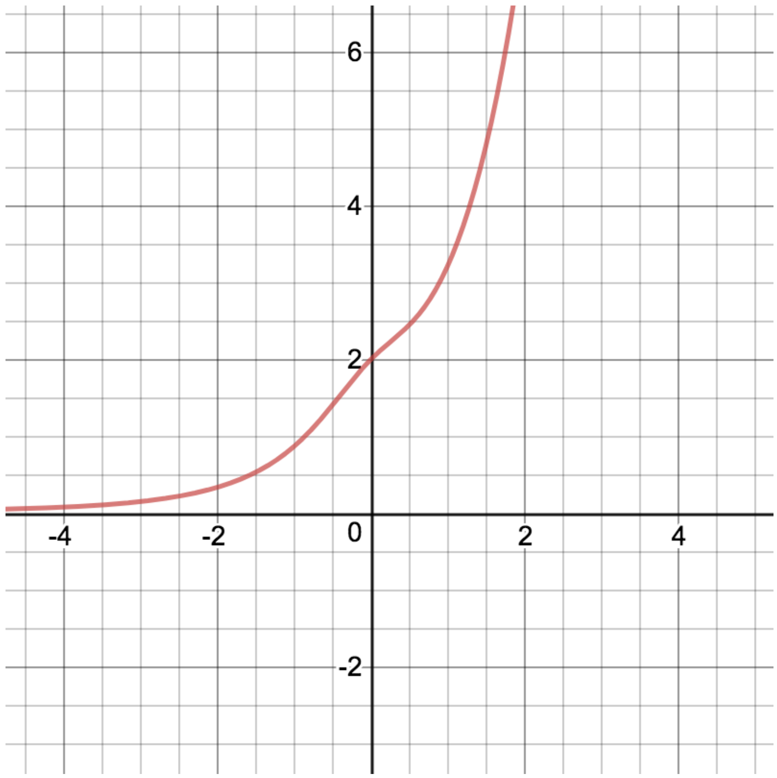

Figure 1: The graph of $f(x) = e^x + 1/(x^2 + 1)$

10. Evaluate the following limits. You may use l'Hopital's rule if it is applicable.

(a) $\lim_{x \to 0} (1 + x)^{2/x}$.

(b) $\lim_{x \to 0} \dfrac{\sin(x^4)}{x^4}$.

(c) $\lim_{x \to \infty} \dfrac{\sin(x)}{x}$.

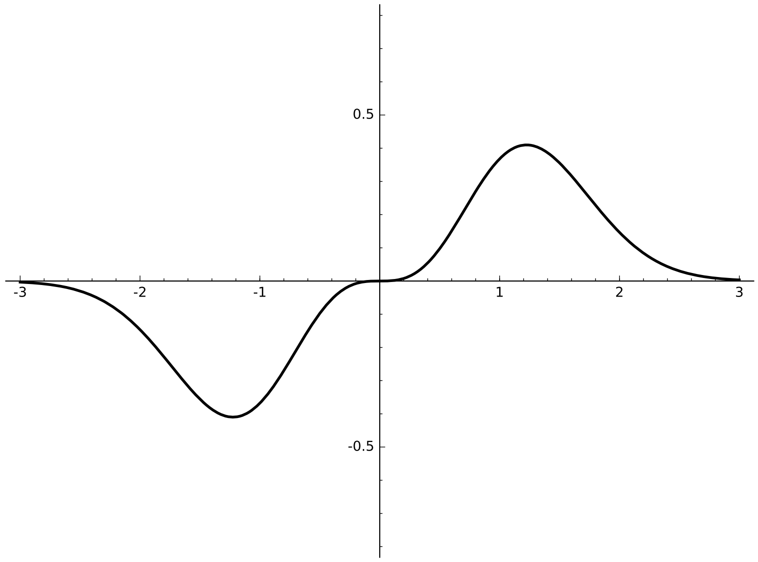

Figure 2: The graph of $f(x) = x^3e^{-x^2}$

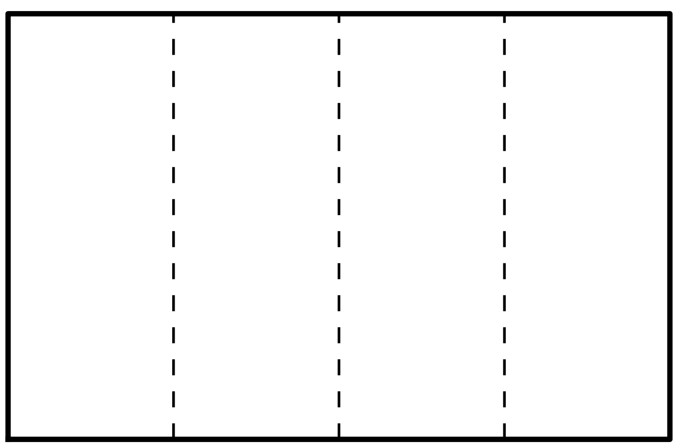

Figure 3: A divided corall