Calc II - Power series problems

1. Find the domain of convergence of each of the following power series:
   (a) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^n}{n!}x^n.$
   (b) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^n}{n}x^n.$
   (c) $\displaystyle\sum_{n=1}^{\infty} n!x^n.$
2. Use Taylor's formula to find the power series representation of $f(x) = \sin(2x)$ at $x = 0$.
3. Use Taylor's formula to find the power series representation of $f(x) = \sin(x)$ at $x = \pi/4$.
4. Let $f(x) = \tan(x)$. Find the third Taylor polynomial of f at $x = 0$.
5. By manipulating some power series that you already know, find power series representations (about zero) of the following functions:
   (a) $f(x) = \sin(2x)$
   (b) $f(x) = \dfrac{x}{x-1}$
   (c) $f(x) = x^2 \ln(1-x)$
   (d) $f(x) = xe^{x^2-1}$
6. In this problem, we're going to derive Leibniz's famous series formula for π:

$$\frac{\pi}{4} = \sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}x^{2n+1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots .$$

   To do so,
   (a) Start with the geometric series formula in terms of x,
   (b) Make the substitution $x \to -x^2$,
   (c) Integrate the result,
   (d) Make the substitution $x \to 1$.