Calc III - Review I

The first exam will be this Friday, February 3. We will spend some time discussing a few of these problems in class on Wednesday but you should work them all out to the best of your ability prior to that. Understanding the problems on this sheet will help you greatly on the exam.

If you think that one of these problems has a mistake, you should try to find the mistake and fix it.

1. Sketch the curve $y^2 + 4z^2 = 4$ in the yz-plane. What does the graph of this equation look like in xyz-space?
2. The equation $x^2 + y^2 + z^2 = 2z$ describes a sphere. What are the center and radius of that sphere?
3. An object moves according to the parametrization $\vec{p}(t) = \langle \sin(12t), -t + \cos(12t) \rangle$.
   (a) Describe the motion determined by $\vec{p}(t)$.
   (b) Parameterize the line tangent to the curve at the point $\vec{p}(\pi/36)$.
4. Possible proofs
   (a) Let $\boldsymbol{v} = \langle x, y \rangle$ and let $\boldsymbol{w} = r\boldsymbol{v}$. Show that $\|\boldsymbol{w}\| = r\,\|\boldsymbol{v}\|$.
   (b) Prove that the two-dimensional dot product is commutative and distributive over vector addition. Is it associative? Why?
   (c) Let $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$ be three dimensional vectors. Prove that $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$.
5. Suppose the force vector $\vec{F} = \langle 2, 1, 1 \rangle$ dispaces an object along the displacement vector $\vec{d} = \langle -1, 0, 1 \rangle$. How much work does the force accomplish in this process?
6. Let $\vec{u} = \langle 3, 1, -1 \rangle$ and $\vec{v} = \langle 1, -3, -2 \rangle$.
   (a) Compute $\mathrm{proj}_{\vec{v}}\vec{u}$, the vector projection of $\vec{u}$ onto $\vec{v}$.
   (b) What is the exact angle between $\vec{u}$ and $\vec{v}$?
7. Find an equation of the plane that is tangent to the sphere $x^2 + y^2 + z^2 = 14$ at the point $(1, 2, 3)$.
8. Find the equation of a plane containing the points $(3, 3, 3)$, $(0, 2, 2)$, and $(-3, 0, 3)$ or explain why no such plane exists.
9. Let $p(t) = (1 + t, -2t, 1 - t)$ and $q(t) = (1 - t, 2t, 1 + 2t)$ be the parameterizations of two lines.
   (a) Find the point of intersection of the two lines.
   (b) Find an equation of the plane that contains the two lines.

10. Suppose we throw an object from the origin with an initial velocity of

$$\vec{v} = \langle 8, 24 \langle,$$

measured in feet per second.

(a) How long is the object in the air?

(b) How far away does the object land?

11. An object moves along the path parameterized by

$$\vec{p}(t) = \langle 2t, \frac{1}{t} \rangle.$$

If it can shoot a laser beam straight ahead, at what time should it shoot to hit the point $(4, 0)$?

12. Suppose the motion of an object is parameterized by $\vec{p}(t) = \langle t^3 + t + t, \sin(t) \rangle$. Write down an integral representing the distance travelled by the object over the time interval $0 \leq t \leq 2$.

13. Match the groovy function below with the groovy picture in figure 1.

(a) $\vec{p}(t) = \langle \cos(3t), \sin(3t) \rangle$.

(b) $\vec{p}(t) = \langle \cos(3t), 0, \sin(3t) \rangle$.

(c) $\vec{p}(t) = \langle \cos(3t), t, \sin(3t) \rangle$.

(d) $\vec{p}(t) = \langle \cos(3t) + t, \sin(3t) \rangle$.

(e) $\vec{p}(t) = \langle 3\cos(3t), \sin(3t) \rangle$.

(f) $\vec{p}(t) = \langle t\cos(3t), t\sin(3t) \rangle$.

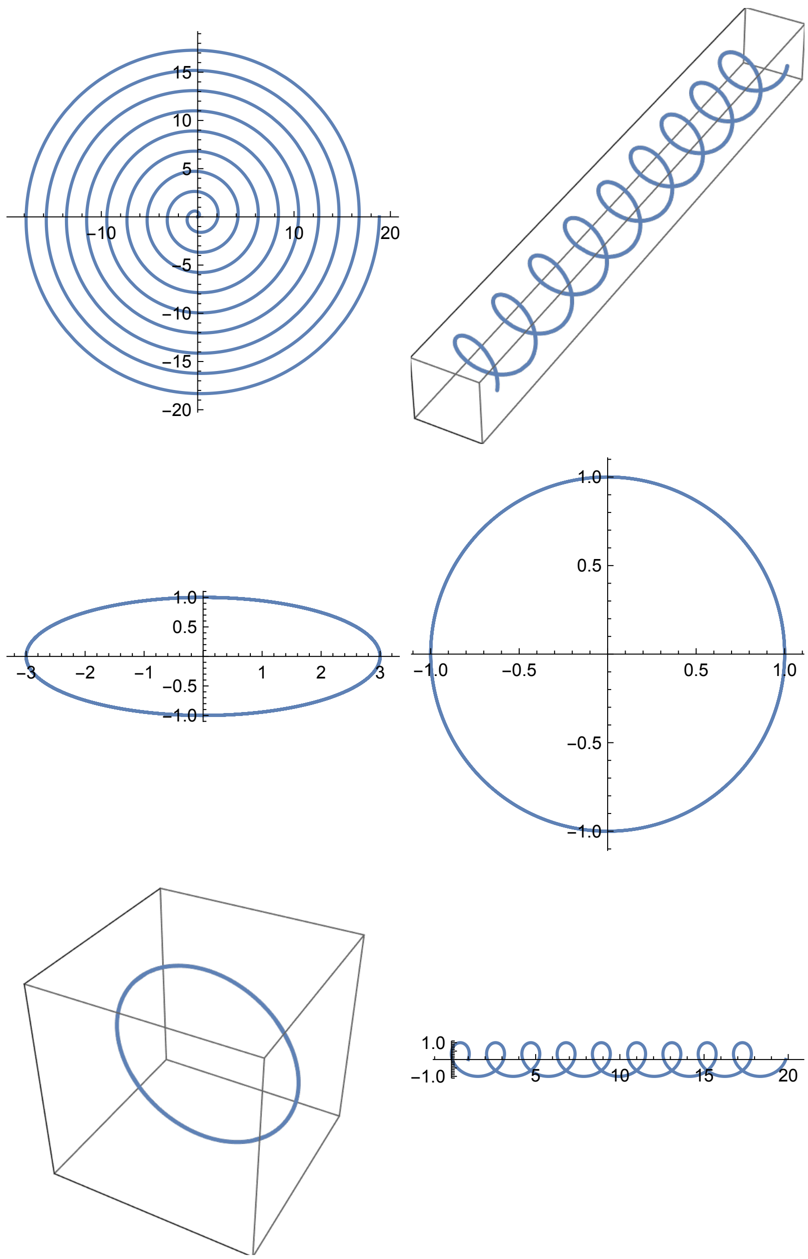

Figure 1: Some groovy pics