Discrete Math - Review review for I

Our first exam will be this coming Thursday, March 6. We will go over this sheet on Tuesday but you should defintely try to figure it out as much as you can before that. Feel free to post solutions on the forum and fire me an email, if I don't give a thumbs up or down.

1. Suppose you draw two cards from a well shuffled deck.
   (a) How many different ways can you draw two Kings?
   (b) How many different ways can you draw a King and then a Queen?
2. Suppose you draw a five card hand from a well shuffled deck. How many different possible hands are there?
3. Use summation notation to expand $(2x^2 - 3y^3)^{100}$.
4. How many ways can I distribute 11 cookies to my 3 kids assuming
   (a) Each kid gets at least one cookie?
   (b) No restriction whatsoever.
5. How many integer solutions to
   $$x_1 + x_2 + x_3 + x_4 = 20$$
   are there when
   (a) Each x_i is positive?
   (b) Each x_i is non-negative?
6. Find a polynomial that generates the following list of integers:
   $$0, 1, 8, 21, 40, 65, 96, 133, 176, \ldots$$
7. Suppose that a_n is defined recursively with
   $$a_0 = 1,\ a_1 = 3,\ \text{and}\ a_n = 6a_{n-1} - 8a_{n-2}.$$
   Find a closed form expression for a_n.
8. Suppose we'd like to travel from the lower left corner to the upper right corner of an 8×5 grid using the smallest possible number of steps. One such path is shown in figure 1.
   (a) Explain how the binomial coefficients can be used to solve this type of problem.
   (b) How many shortest paths of this type are there for a 20×20 grid?

   I would expect your answer to be expressed in terms of binomial coefficients.
9. Suppose I'd like to cover a row of squares with square tiles and rectangular tiles of aspect ratio 2. I'm allowed to cover one square with a square tile and two squares with a rectangular tile. Figure 2a shows a row of length 10 and figure 2b shows such a row covered by 4 squares and 3 rectangles. More generally, though, I'd like to be able to cover a row of length n for any positive integer n.
   (a) Explain how the Fibonacci numbers arise in this process.
   (b) How many ways are there to cover a row of length 5?
   (Express your answer as a simple integer.)
   (c) How many ways are there to cover a row of length 20?
   (Express your answer as a Fibonacci number.)
10. Use induction to show that
    $$\sum_{k=1}^{n}(6k - 1) = 3n^2 + 2n.$$

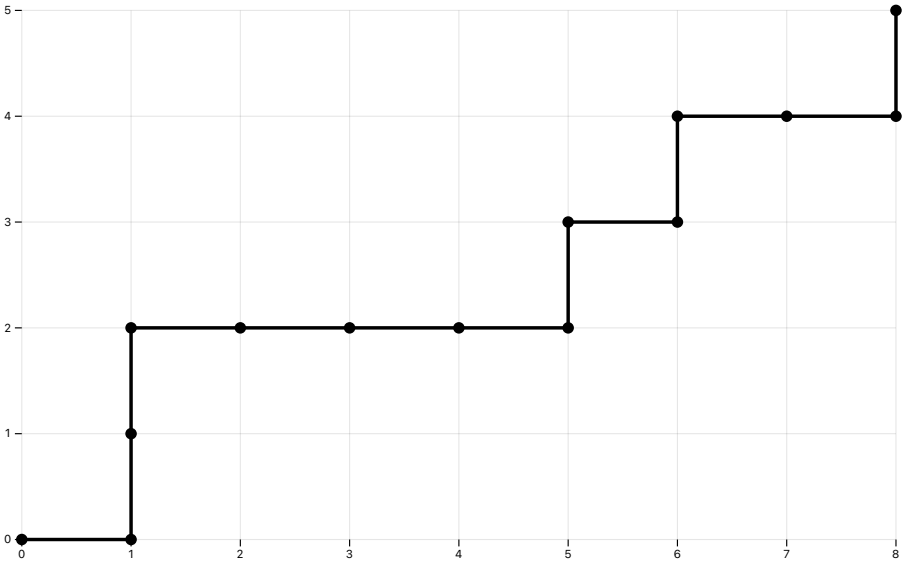
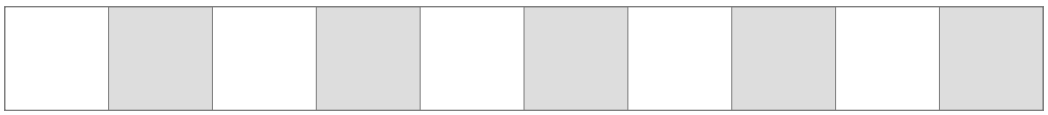
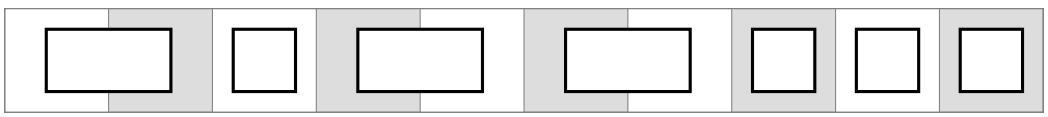


Figure 1: A shortest path through an 8×5 grid



(a) An empty row of length 10



(b) A row of length 10 covered with squares and dominos

Figure 2: Two rows